Lecture 7

(iii) Electric Field due to a concentric spherical shells

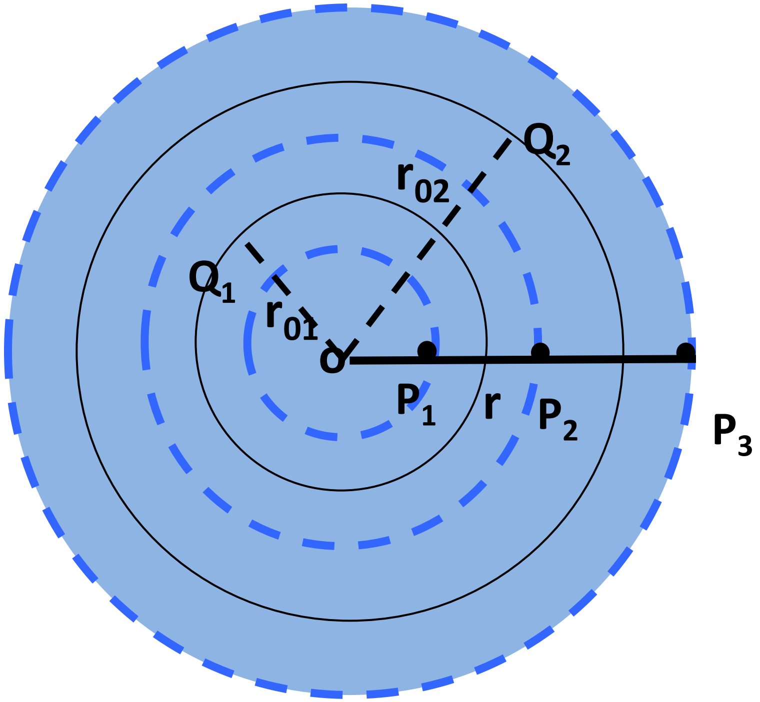
Two concentric spherical shell of radii r_{01} and r_{02} meters bearing charges Q_1 and Q_2

(a) Field inside the inner shell $r < r_{01}$

Charge inside the shell of radius r is zero

$\therefore$ Electric field intensity at P_1

$$E = 0$$



(b) Field between the shells $r_{01} < r < r_{02}$

Charge inside the shell of radius r is Q_1

$\therefore$ Electric field intensity at P_2

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r^2}$$

(c) Field outside both shells $r > r_{02}$

Charge inside the shell of radius r is $Q_1 + Q_2$

$\therefore$ Electric field intensity at P_3

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 + Q_2}{r^2}$$

(ii) Electric Field due to uniformly charged cylindrical conductor of infinite length

•Let there be a cylindrical conductor of infinite length and radius R which is uniformly charged with charge per unit length λ.

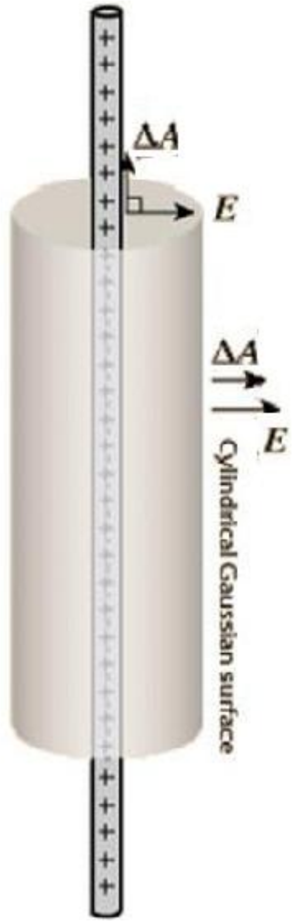
•As the material of the cylinder is conducting, total charge resides on the outer surface of the cylinder and there is no charge with in the cylinde

•We are to find the electric field intensity at a point P, at distance r from the axis

λ = charge per unit length

- **When the point P is out side the cylinder**
- **When the point P is on the cylinder**
- **When the point P is inside the cylinder**

(i) *Electric Field strength due to infinite line charge*



- Consider a coaxial cylinder of length h and radius r as the Gaussian surface.

- Since the cylinder is uniformly charged, therefore the electric field intensity at any point on the curved surface of Gaussian surface will be the same and directed normally to the surface in an outward direction.

- Thus the electric flux contribution through the plane surfaces of the Gaussian surface will be zero.

- Only the electric flux through the curved surfaces of the Gaussian surface will contribute.

➢ There are three surfaces to consider. The upper (A_1) and lower(A_2) circular surfaces have normals are perpendicular to the electric field, thus contribute zero to the flux.

The electric flux due to each plane faces $\oint_{A_1, A_2} E \cdot dA = 0$

The electric flux due to curved surface $\oint_{A_3} E \cdot dA = E.2\pi\, rh$

According to Gauss's theorem, the electric flux through the Gaussian surface

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$\varepsilon_0 \oint E \cdot dA = q_{enc}$$

The charge inside the Gaussian surface = charge on the length h of the cylinder = λh

$$\varepsilon_0 E(2\pi\, rh) = \lambda h$$

$$E = \frac{\lambda}{2\pi\varepsilon_0\, r} \qquad \text{N/C}$$

Thus the electric field strength is inversely proportional to r.

Case(ii) When the point P is on the cylinder *(r = R)*

$$E = \frac{\lambda}{2\pi\varepsilon_0 R} \quad \text{N/C}$$

Case(iii) When the point P is inside the cylinder *(r < R)*

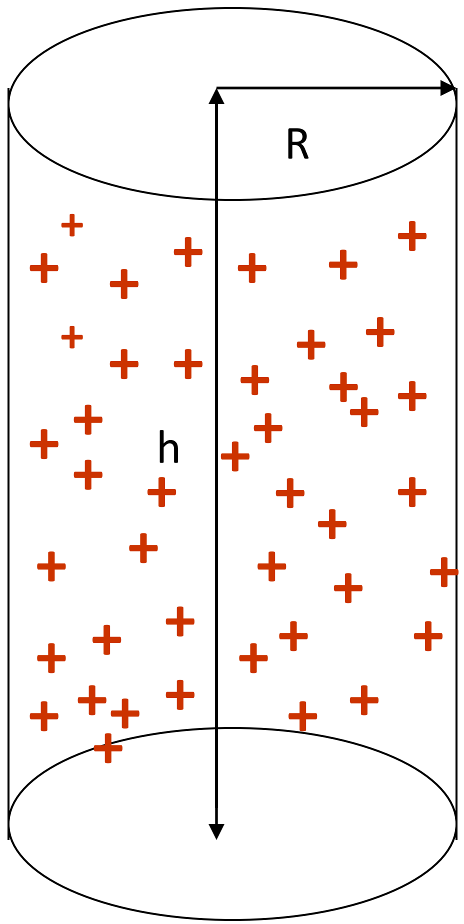
- For the conducting cylinder, the total charge lies only on its outer surface.
- There is no charge within it.
- Hence the electric flux through the Gaussian surface will be zero

$$E = 0$$

(ii) Electric Field due to uniformly charged non-conducting cylinder

Let us consider that electric charge is uniformly distributed within an infinite cylinder of radius R

If λ is the charge per unit length and ρ is the volume charge density, then for a cylinder of length h and radius R



$$\pi R^2 h\rho = \lambda h$$

$$\rho = \frac{\lambda}{\pi R^2}$$

Case (i) When point P lies outside the charge distribution i.e. r >R

Due to symmetry the electric field strength E_o is every where normal to the curved surface

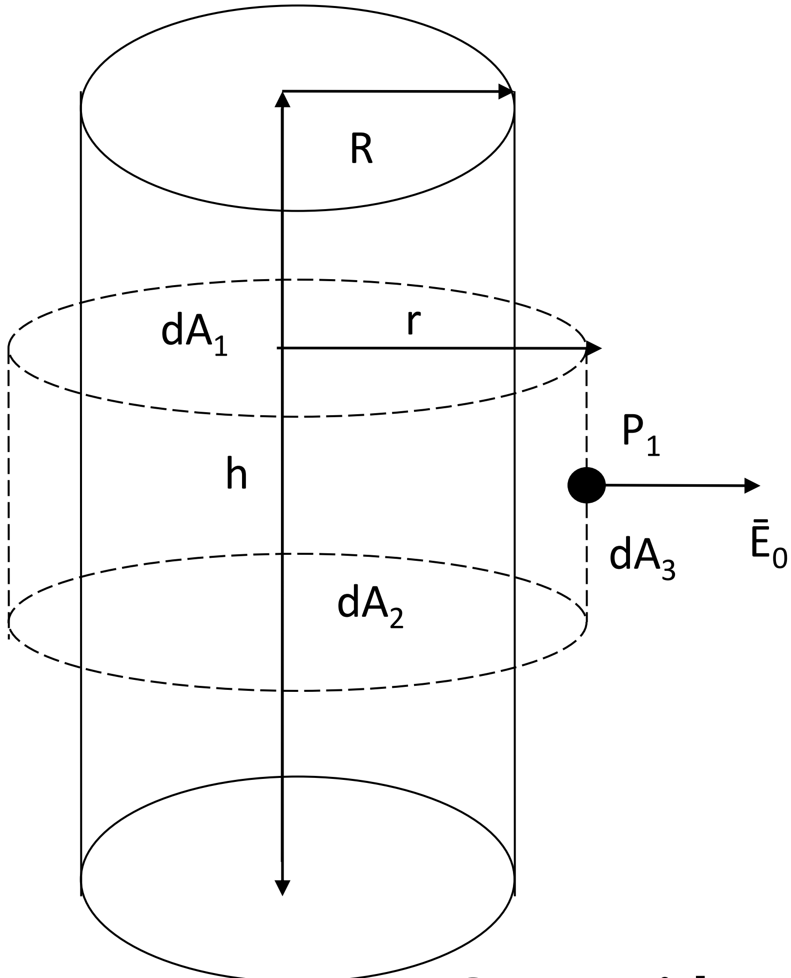
Further E_0 being parallel to two flat bases of the cylindrical surface considered, the contribution to electric flux due to circular surfaces is zero.

∴ Electric flux through the cylindrical surface assumed

$$\int_A E \bullet ds = \int_{A_1} E \bullet dA_1 + \int_{A_2} E \bullet dA_2 + \int_{A_3} E \bullet dA_3$$

$$\int_A E \bullet dA = 0 + 0 + \int_{A_3} E_0 dA_3 \cos 0^0$$

$$\int_S E \bullet dA = E_0 \int dA_3 = E_0 \cdot 2\pi r h$$



$Q_{encl} = \lambda h$

According to Gauss's theorem

$$\int_s E \bullet ds = \frac{Q_{enc.}}{\varepsilon_0}$$

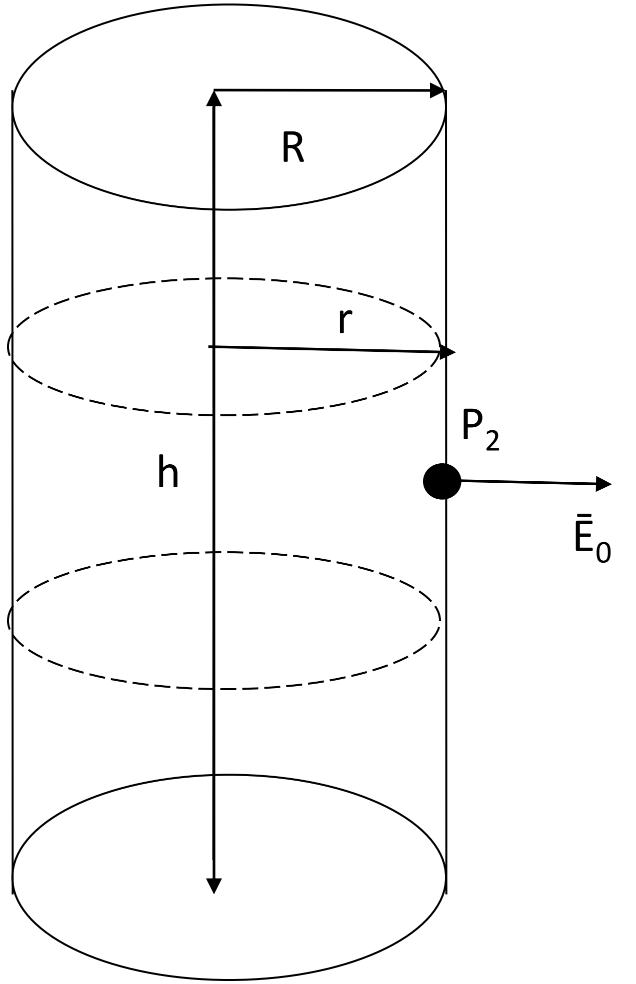
$$E_0 \cdot 2\pi r h = \frac{\lambda h}{\varepsilon_0}$$

$$E_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\lambda}{r}$$

Thus the electric field strength due to a uniform infinite cylindrical charge at any point outside the charge distribution is same as that due to an infinite line charge.

Case (i) When point lies on the surface of charge distribution i.e. r = R

In this case according to Gauss's theorem



$$\int_A E \bullet dA = \frac{Q_{enc.}}{\varepsilon_0}$$

$$E_s \cdot 2\pi R h = \frac{\lambda h}{\varepsilon_0}$$

$$E_s = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\lambda}{R}$$

Case (i) When point lies inside the charge distribution i.e. r < R

According to Gauss's theorem

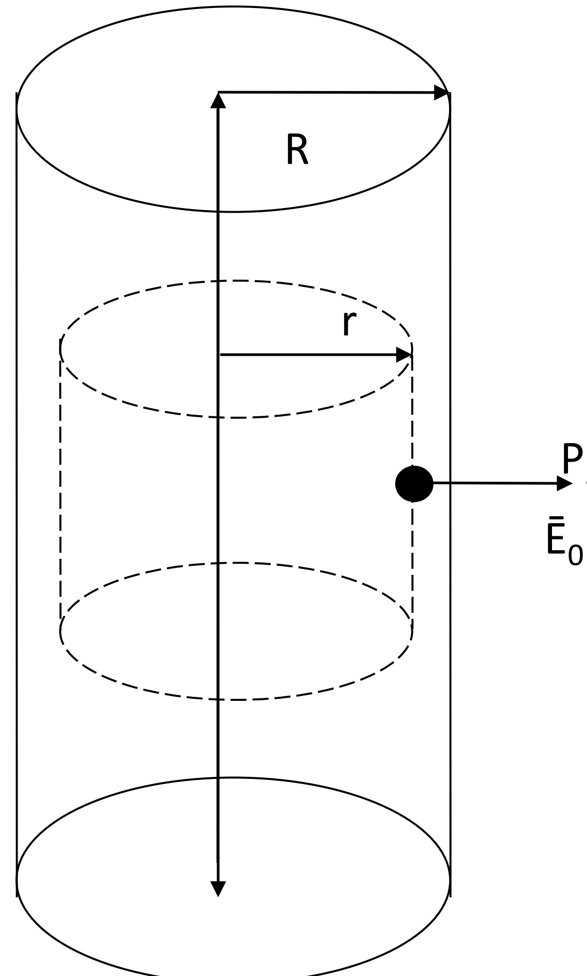
$$\int_s E \bullet dA = \frac{Q_{enc.}}{\varepsilon_0}$$

$$E_{in} \cdot 2\pi r h = \frac{\pi r^2 h \rho}{\varepsilon_0}$$

$$\left(\rho = \frac{\lambda}{\pi R^2 h} \right)$$

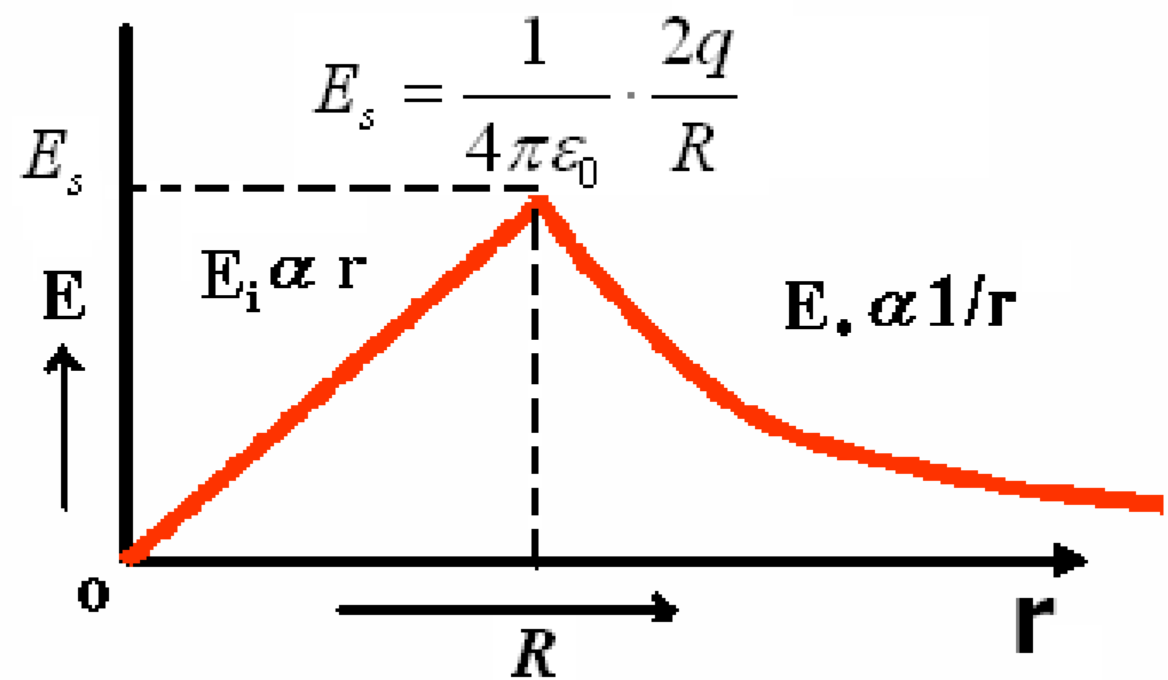
$$E_{in} = \frac{\pi r^2 h \rho}{\varepsilon_0 \cdot 2\pi r h} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\lambda r}{R^2}$$

$$E_{in} \propto r$$



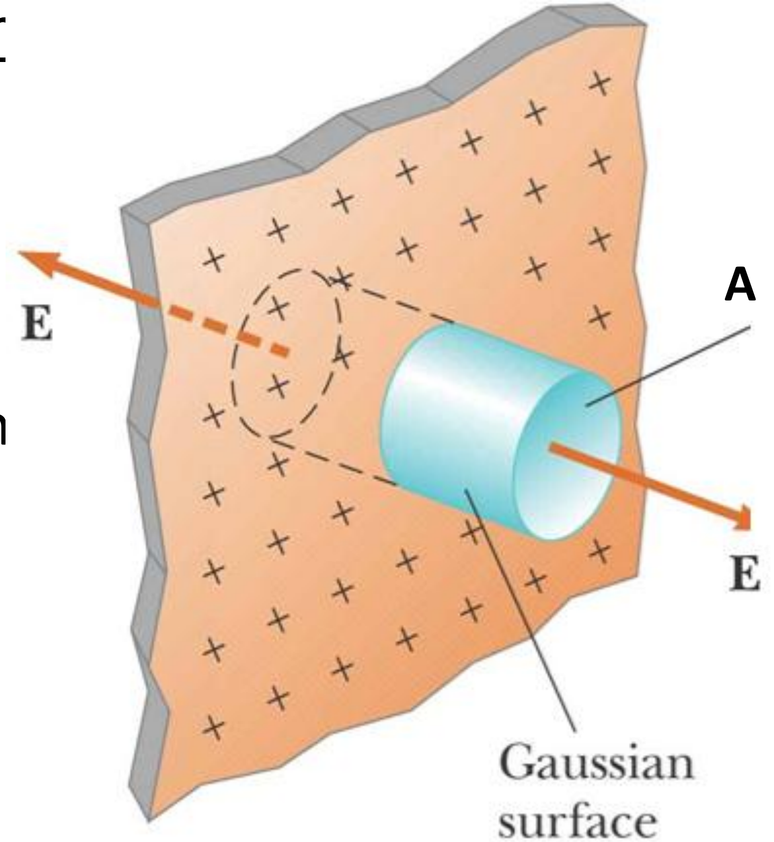
$$\rho = \frac{\lambda}{\pi r^2 h}$$

The variation of electric field strength E with the distance r from the axis of the uniform infinite cylindrical charge distribution



Electric field strength due to an infinite non-conducting flat sheet of charge

- Assume that we have a thin, infinite r sheet of positive charge.



• The charge density in this case is the charge per unit area,

• Consider an imaginary cylindrical Gaussian surface inserted into sheet.

•The charge enclosed by he surface is q=σA

•From symmetry, we can see that the electric field will be perpendicular to the surface of the sheet

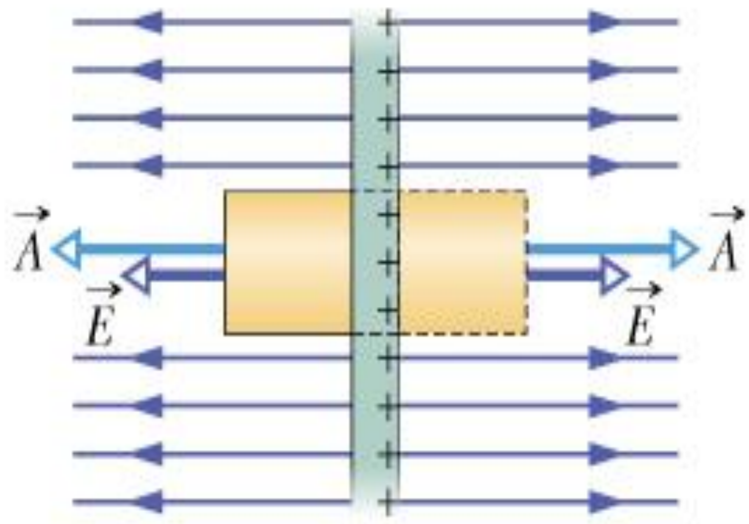
➢Due to symmetry electric field strength E is normal outwards at the points on the two plane surfaces and parallel to the curved surface.

- There is no flux from the curved surface of the cylindrical
- There is equal flux out of both end caps

$\therefore$ Total electric flux = EA + EA = 2EA

According to Gauss theorem

$$\oint E \bullet dA = \frac{Q_{encl}}{\varepsilon_0}$$



$$2EA = \frac{1}{\varepsilon_0}(\sigma A)$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Thus electric field strength due to an infinite flat sheet of charge is independent of the distance.

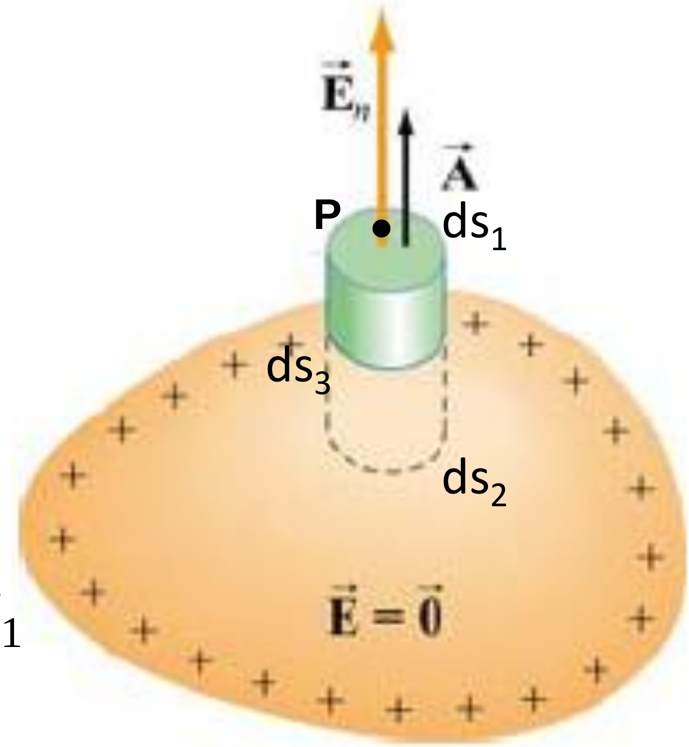
Electric field strength just outside a charged conductor

Consider a small Gaussian cylindrical box as drawn in fig.

Let the surface charge density on the surface of the conductor be σ

Let the area of each base is --- A

The Electric field inside the conductor is zero



Total electric flux

$$\int_s E \bullet ds = \int_{s_1} E \bullet ds_1 + \int_{s_2} E \bullet ds_2 + \int_{s_3} E \bullet ds_3$$

$$\int_s E \bullet ds = \int_{s_1} E \bullet ds_1 = \int_{s_1} E ds_1 \cos 0^0 = \int_{s_1} E ds_1$$

$$= EA$$

Charge enclosed by the cylinder **$Q_{encl} = \sigma.A$**

According to Gauss theorem

$$\oint E \bullet ds_1 = \frac{Q_{encl}}{\varepsilon_0}$$

$$E.A = \frac{1}{\varepsilon_0}(\sigma A)$$

$$E = \frac{\sigma}{\varepsilon_0}$$

The electric field strength at any point close o the surface of a charged conductor of any shape is equal to 1/ε_0 times the surface charge density σ