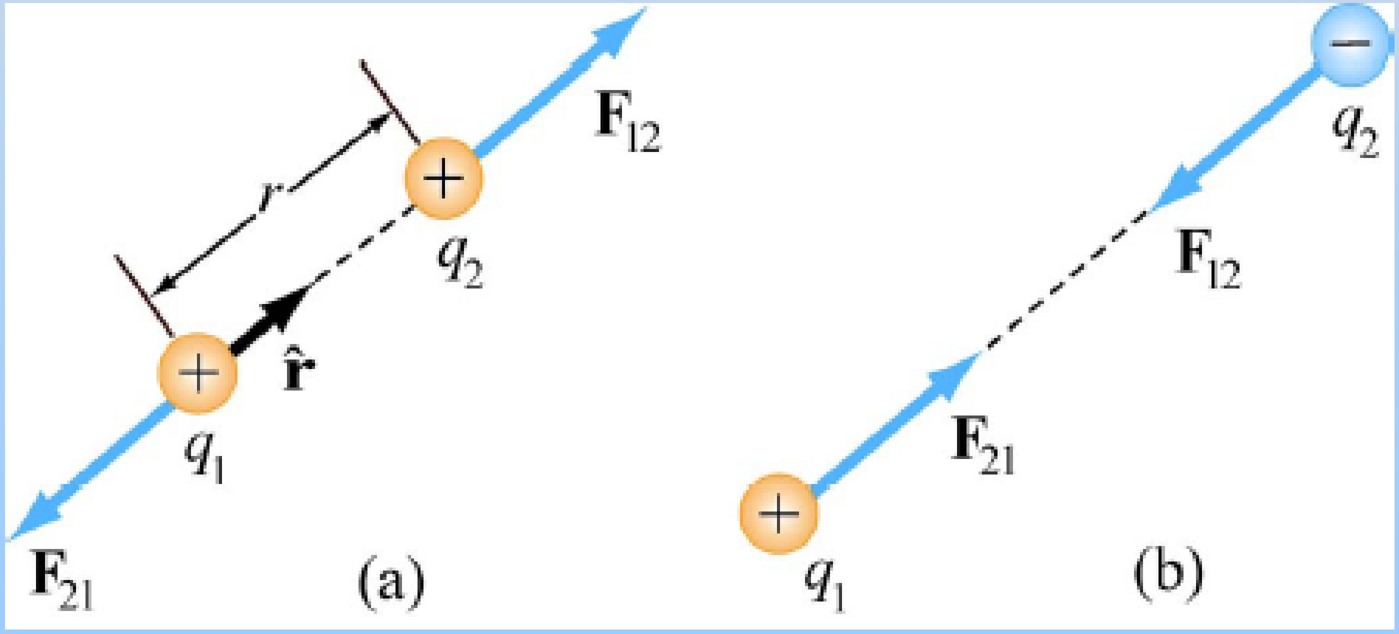
Lecture 5

Electric Force

The electric force between charges q_1 and q_2 is

(a) repulsive if charges have same signs

(b) attractive if charges have opposite signs



Like charges repel and opposites attract !!

Coulomb's Law

- Coulomb determined
  - Force is attractive if charges are opposite sign
  - Force proportional to the product of the charges q_1 and q_2 along the lines joining them
  - Force inversely proportional square of the distance

$$|F_{12}| \propto |Q_1|\ |Q_2| \,/\, r_{12}^2$$

$$|F_{12}| = k\ |Q_1|\ |Q_2| \,/\, r_{12}^2$$

k is normally expressed as $k = 1/4\pi\varepsilon_0$

where is the permittivity of free space

k = 8.987.5x10^9 Nm^{2}C^{-2}

Coulomb's Law

The equation for the magnitude of the Coulomb force between two point charges Q_1 and Q_2 in a vacuum is given by

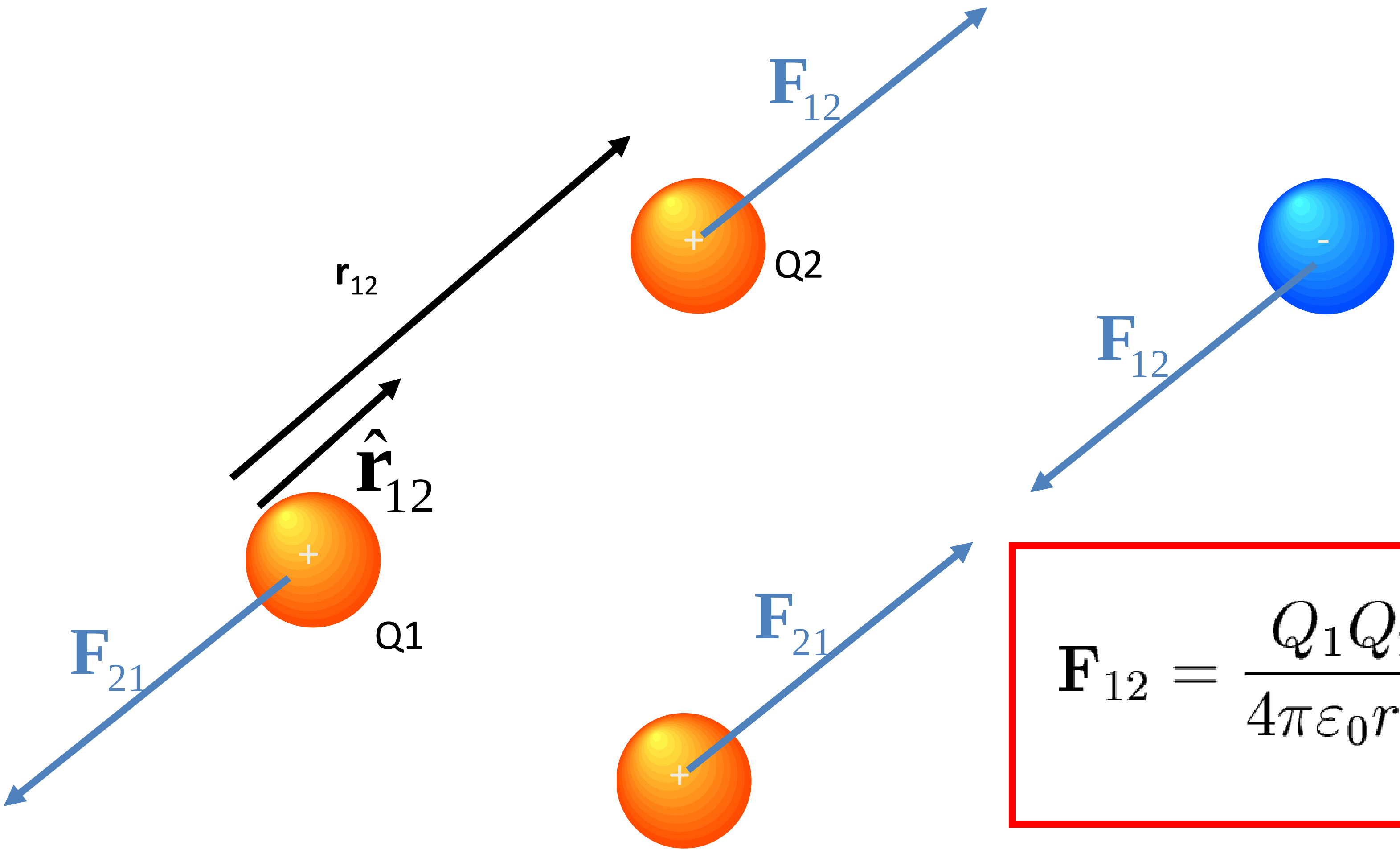
$$|\mathbf{F}_{12}| = \frac{|Q_1 Q_2|}{4\pi\varepsilon_0 r_{12}^2}$$

where

$\lvert Q_1 \rvert$	is	the magnitude of the charge Q_1 in coulombs (C)
$\lvert Q_2 \rvert$	is	the magnitude of the charge Q_2 in coulombs (C)
$\mathbf{F}_{12}$	is	the electrical force acting on the charge Q_1 due to charge Q_2 in newtons (N)
r_{12}	is	the distance between the point charges Q_1 and Q_2 in metres (m)
ε_0	is	the permittivity of free space in $C^2\ N^{-1}\ m^{-2}$
$\frac{1}{4\pi\varepsilon_0}$	is	the Coulomb constant in $N\ m^2\ C^{-2}$.

The direction of the force $\mathbf{F}_{12}$ is determined by the sign of the charges; the force is attractive if the charges have opposite signs, and repulsive if the charges have the same sign.

Vector form of Coulomb's Law



$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r_{12}^2}\hat{\mathbf{r}}_{12},$$

$$\varepsilon_0 = \frac{1}{36\pi \cdot 10^9} = 8.85 \cdot 10^{-12} \quad \frac{\mathrm{C}^2}{\mathrm{m}^2 \cdot \mathrm{N}}$$

Example 8

Two point charges Q_1 = 50 μc and Q_2 = 10 μc located at (-1,1,-3) m in (3,1,0) m respectively. Find the force on Q_1

$$\overrightarrow{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\widehat{r}_{21}$$

$r = -4a_x - 3a_z$

$$\widehat{r} = \frac{-4a_x - 3a_z}{5}$$

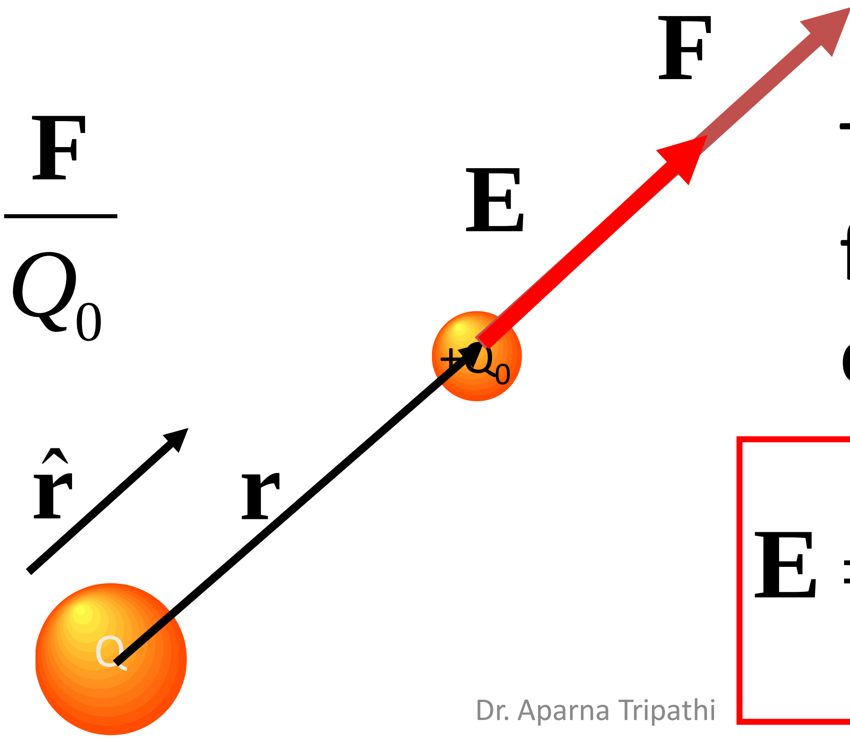
$$\overrightarrow{F}_1 = \frac{(50\times 10^{-6})(10^{-5})}{4\pi\left(\frac{10^{-9}}{36\pi}\right)(5)^2}\left(\frac{-4a_x - 3a_z}{5}\right) = (.18)(-0.8a_x - 0.6a_z)N$$

$$\overrightarrow{F}_1 = (0.144a_x - 0.108a_z)N$$

Electric Field

Electric Field E is defined as the force acting on a test particle divided by the charge of that test particle

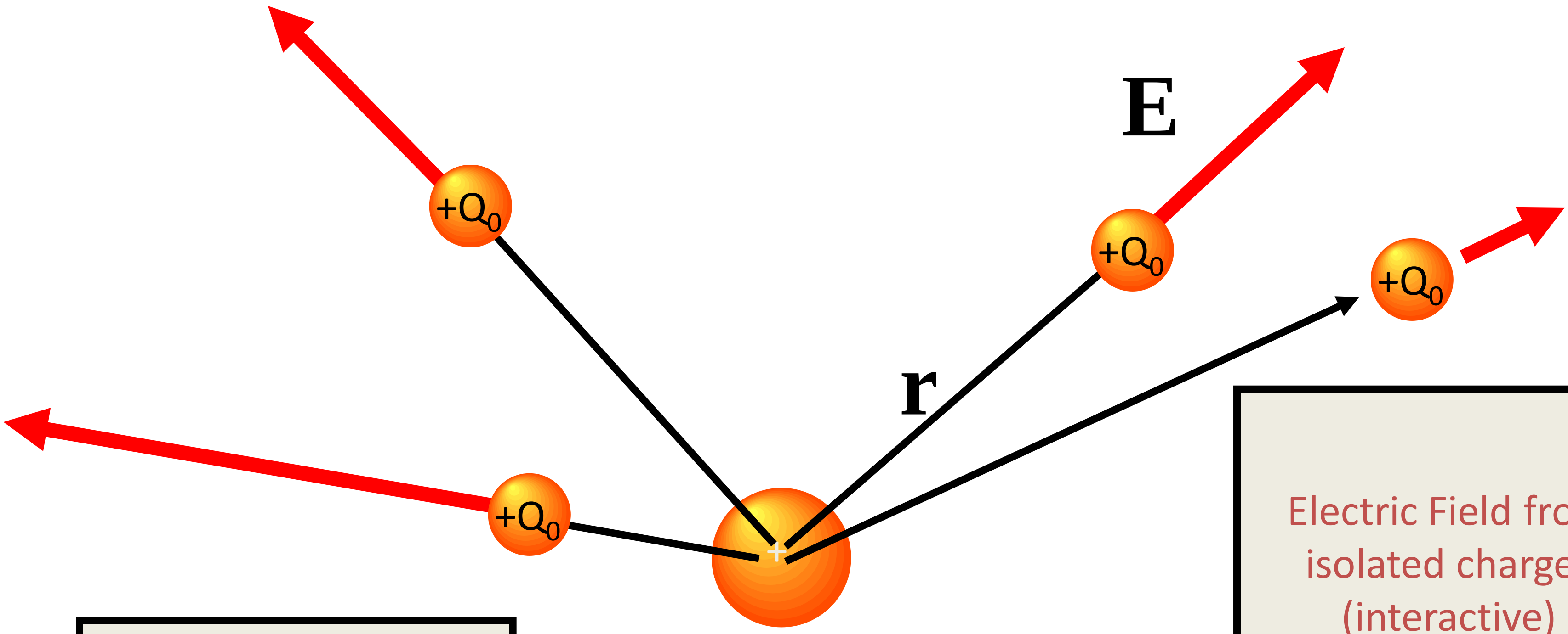
$$\mathbf{E} = \frac{\mathbf{F}}{Q_0}$$



Thus Electric Field from a single charge is

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}|^2}\hat{\mathbf{r}}$$

Electric Field of a single charge



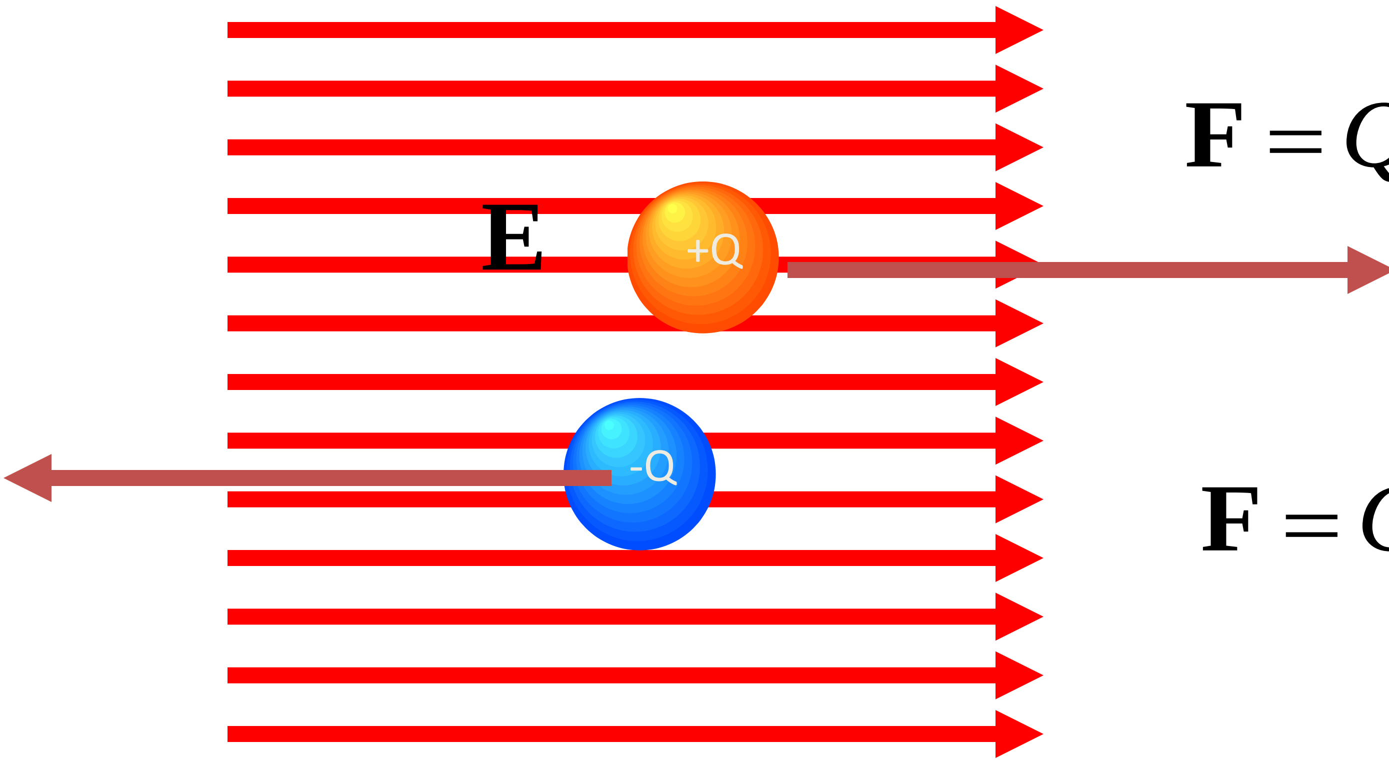
Electric field from test particles

Electric Field from isolated charges (interactive)

Note: the Electric Field is defined everywhere, even if there is no test charge is not there.

Charged particles in electric field

Using the Field to determine the force



$$\mathbf{F} = Q\mathbf{E}$$

$$\mathbf{F} = Q\mathbf{E}$$

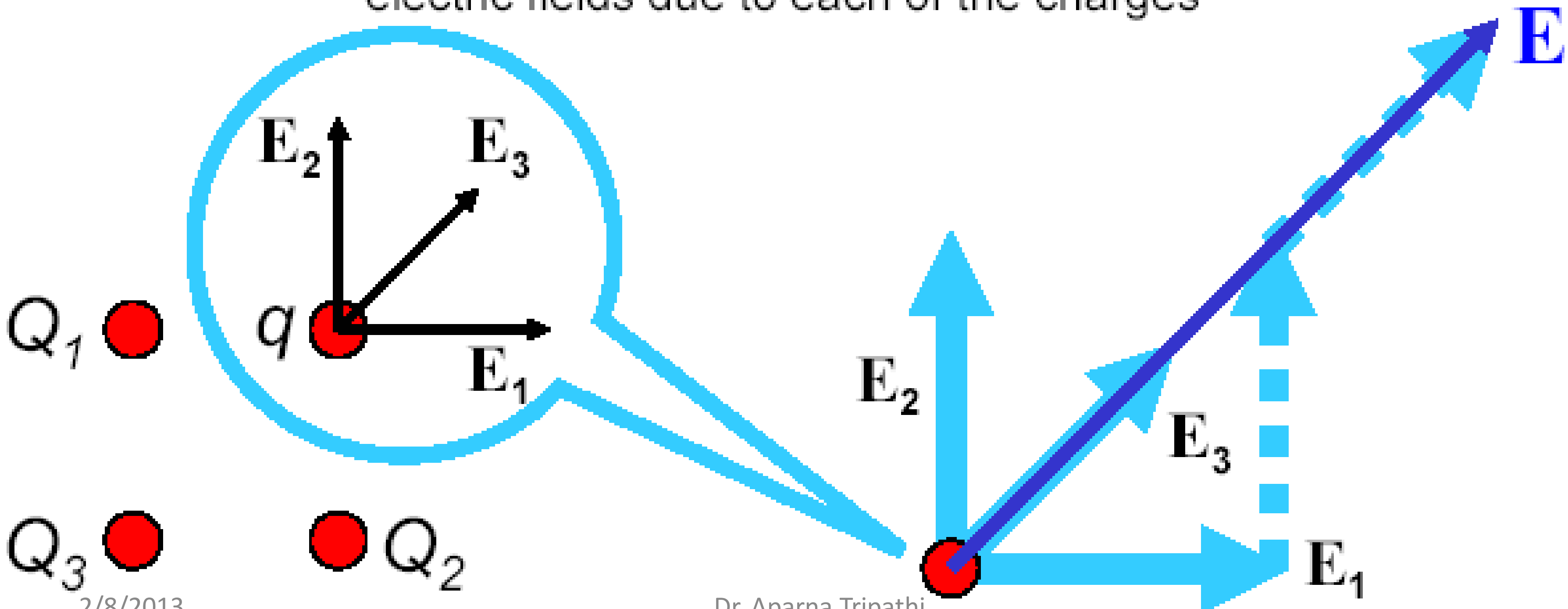
ELECTRIC FIELD FROM MULTIPLE CHARGES: SUPERPOSITION PRINCIPLE

Superposition of forces: $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots$

Therefore, for the electric field intensity

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \ldots$$

Electric field due to multiple charges Q_1, Q_2 , Q_3 , etc is a **vector** sum of the electric fields due to each of the charges



Electric Flux

•Flux is a measure of the number of field lines passing through an area

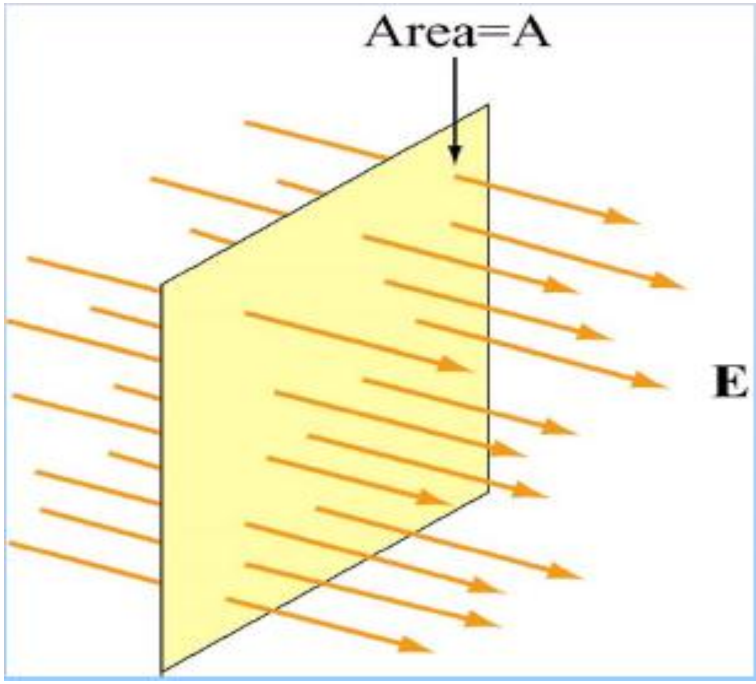
•Electric flux is the total number of Electric lines of forces passing normally through a surface in an electric field is called electric flux.

•It is denoted by the letter ϕ.

•Total Electric flux passing through

the total surface

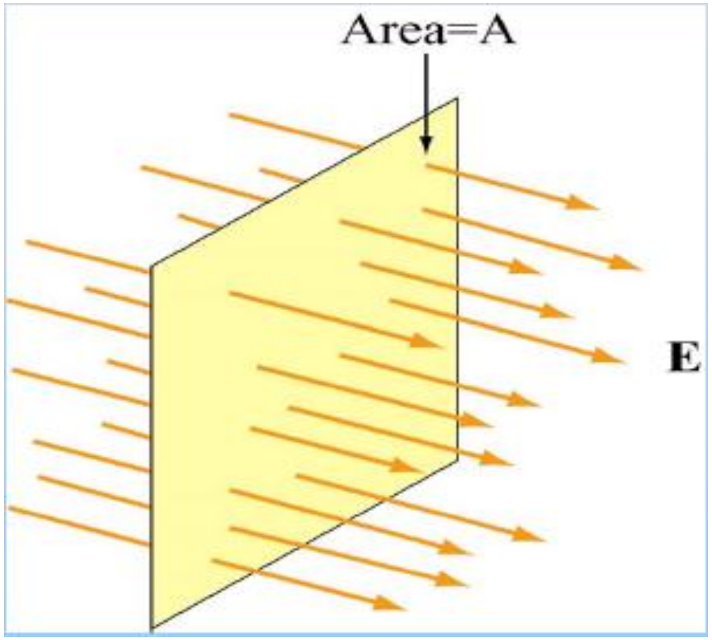
$$\Phi_E = \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$



Electric field lines passing through a surface of area *A,* whose normal makes an angle θ with the field.

$$Electric\ Flux = \Phi = (E\cos\theta)A = \vec{E}\cdot\vec{A}$$

Case I: E is constant vector field perpendicular to planar surface S of area A



$$\Phi_E = \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\text{Electric Flux} = \Phi = (E\cos\theta)A = \vec{E} \cdot \vec{A}$$

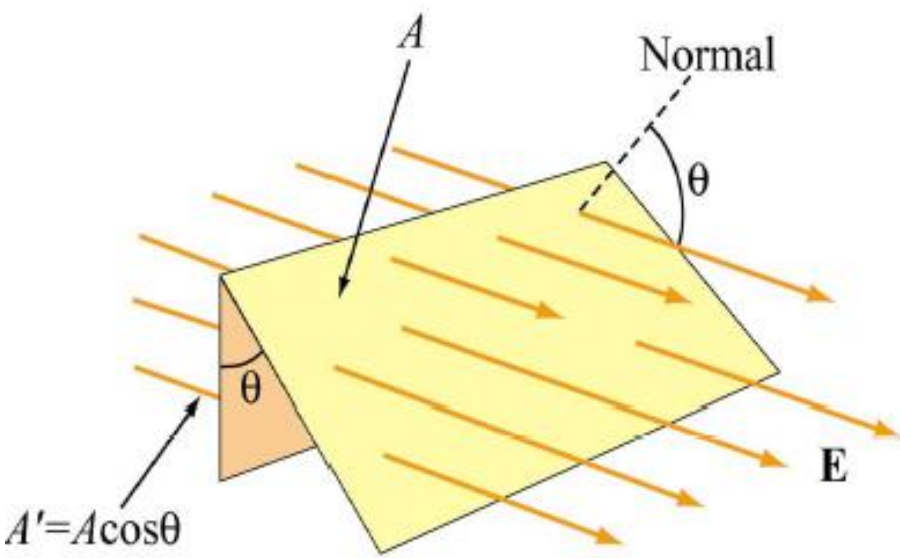
$$\bar{E} \,||\, \bar{A} \Rightarrow \theta = 0$$

$$\Phi_E = +EA$$

Case II: E is constant vector field directed at angle θ to planar surface S of area A

Electric field lines passing through a surface of area *A* whose normal makes an angle θ with the field.

Ē И Ā ⇒ θ ≠ 0



A′ = A cos θ

Where A′ is the perpendicular area to the field E

$$\Phi_E = \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\Phi_E = EA\cos\theta$$

Units: Nm^2/C in SI units, **the electric flux is a SCALAR quantity**

The value of electric flux is +ve if lines of forces are diverging

The value of electric flux is -ve if lines of forces are converging

Example 9

Find the flux of the vector field $\mathbf{A} = \mathbf{a}_r / r^2$ out of the sphere $r = a$, $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$. We have

$$\text{Flux} = \oiint_s \mathbf{A}\Big|_s \cdot d\mathbf{s}$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{r^2} \mathbf{a}_r \Big|_{r=a} \cdot \mathbf{a}_r a^2 \sin\theta \, d\theta \, d\phi,$$

$$\text{Flux} = \int_0^{\pi} \int_0^{2\pi} \sin\theta \, d\theta \, d\phi = 4\pi.$$