Electromagnetism

Lecture 2

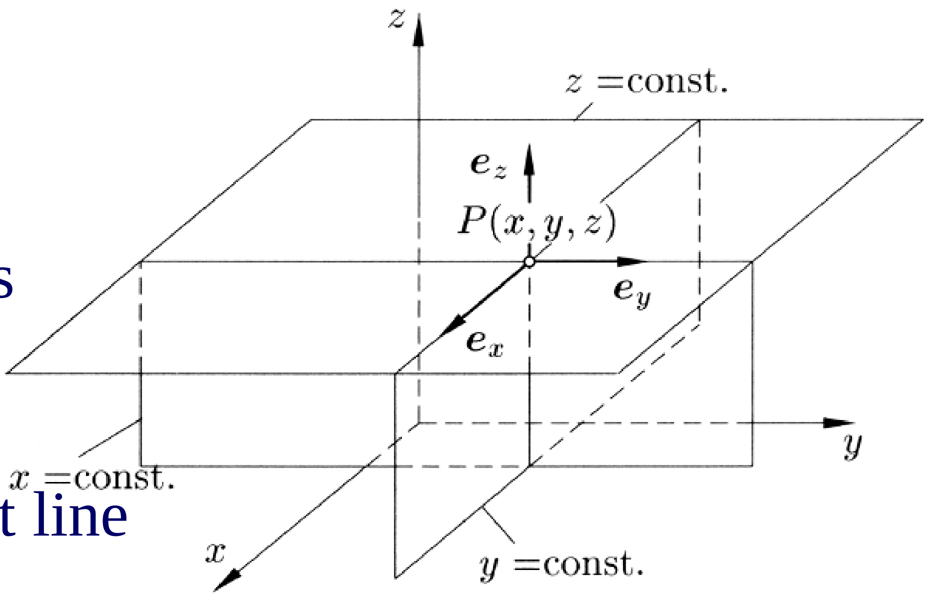
Coordinate System

Different Coordinate Systems

- Cartesian (Rectangular) Coordinate System
- Cylindrical Coordinate System
- Spherical Coordinate System

Cartesian Coordinate System

- Cartesian (Rectangular) Coordinate System is defined by a set of three mutually perpendicular planes



- The point where all the three planes intersect is known as the origin.
- Pair of planes intersect in a straight line
- Hence three planes define a set of three straight lines that are called coordinate axis. Which are denoted by x,y,and z.

Coordinates x, y, z

- The value of x,y and z at origin is (0,0,0)

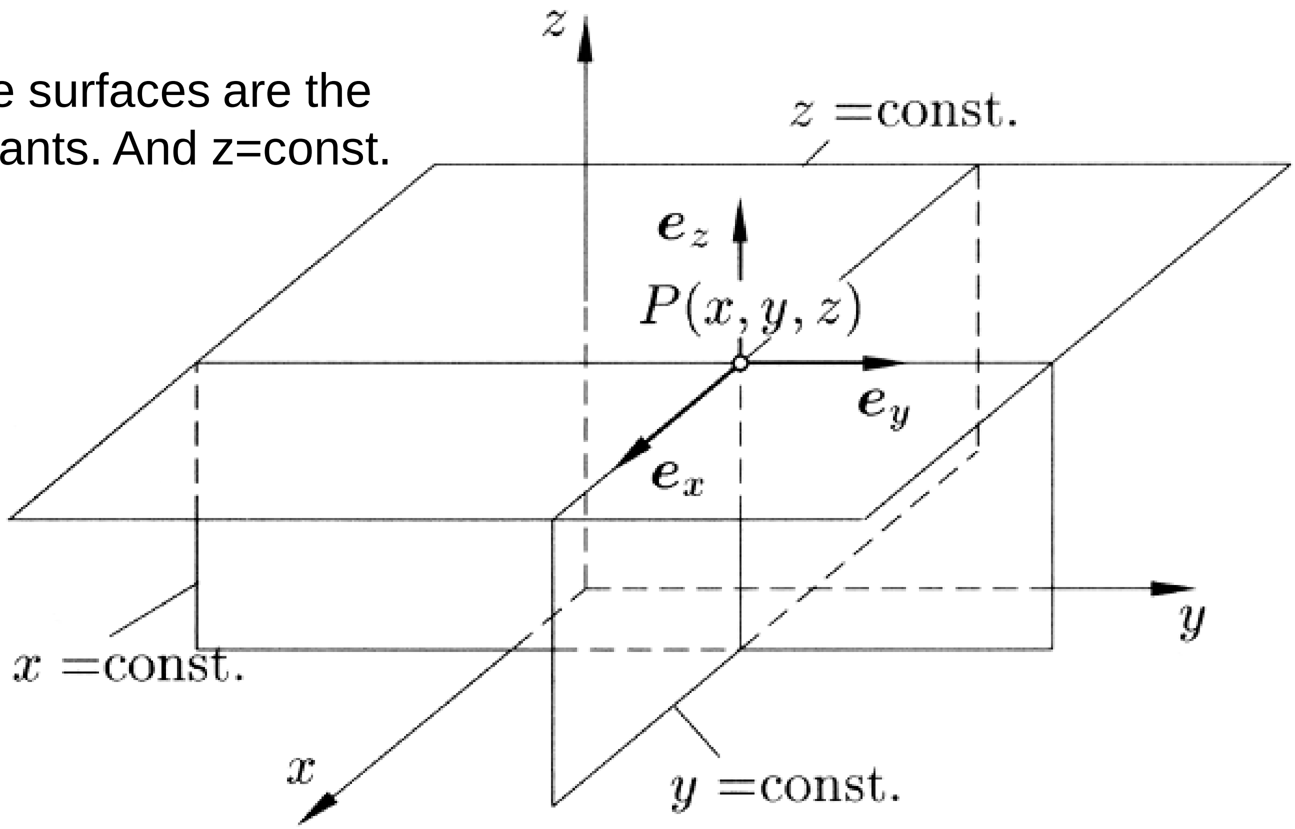
Cartesian Coordinate System (cont.)

• A point is also defined by the intersection of three orthogonal surfaces.

• In cartesian coordinates the surfaces are the infinite planes x= const., y=cants. And z=const.

Limits

$$-\infty < x < \infty$$
$$-\infty < y < \infty$$
$$-\infty < z < \infty$$



• yz plane - x constant
• zx plane - y constant
• xy plane - z constant

Unit vectors have fixed directions, independent of the location of point P

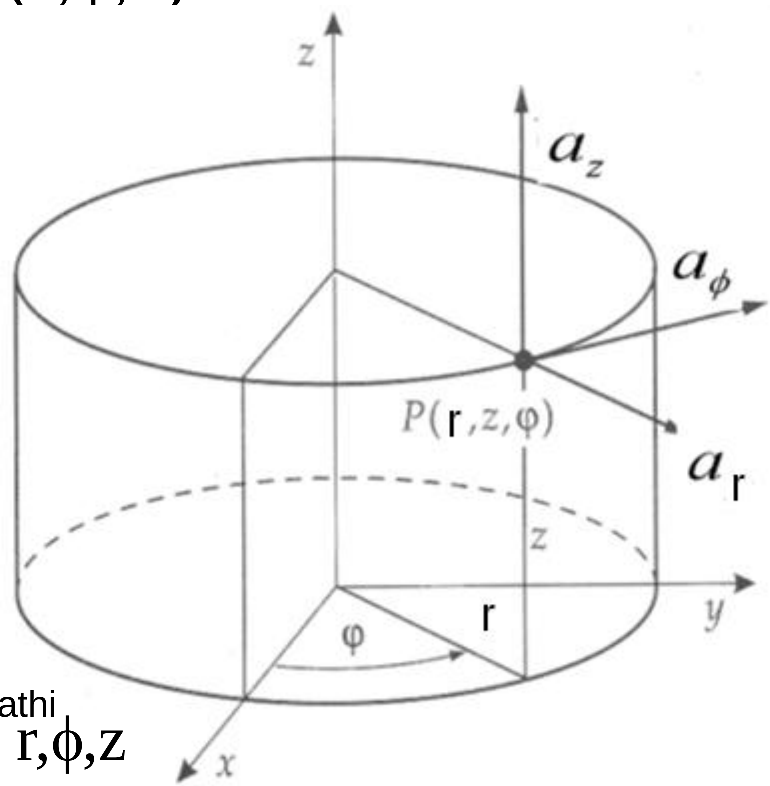
CYLINDRICAL COORDINATES

- Very convenient when dealing with problems having cylindrical symmetry.

• Cylindrical coordinate system involves set of three mutually perpendicular surfaces.

•The three surfaces are one cylindrical and two planes.

•A point P in cylindrical coordinate is (r,ϕ,z) where



r : is the radius of the cylinder; radial displacement from the z-axis

Φ: *azimuthal* angle or the angular displacement from x-axis

z : vertical displacement z from the origin (as in the cartesian system).

Coordinates r,ϕ,z

CYLINDRICAL COORDINATES

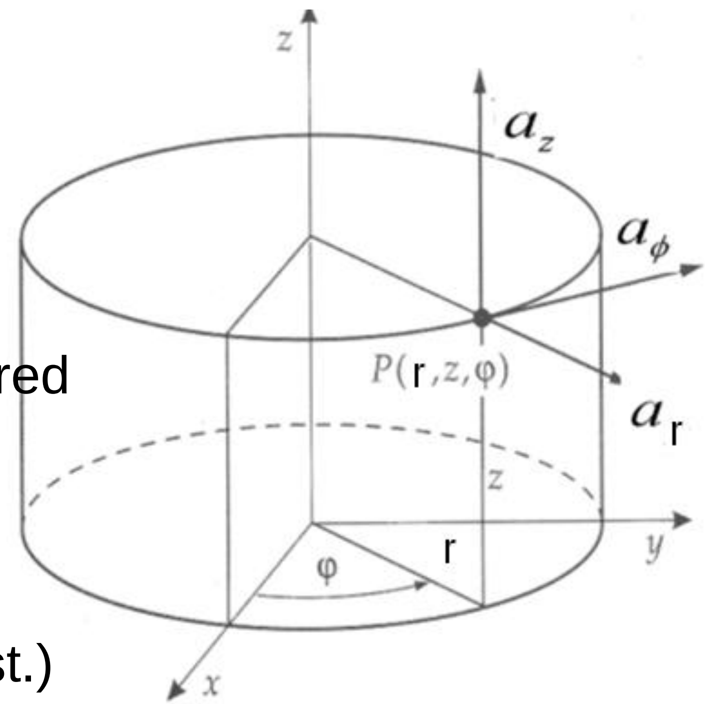
- The range of the variables are

  $0 \leq r < \infty, \quad 0 \leq \Phi < 2\pi, \quad -\infty < z < \infty$

- vector $\vec{A}$ in cylindrical coordinates can be written as (A_r, A_ϕ, A_z) or $A_r a_r + A_\phi a_\phi + A_z a_z$

- The magnitude of $\vec{A}$ is

$$|\vec{A}| = \sqrt{A_r^{2} + A_\phi^{2} + A_z^{2}}$$



Cylindrical coordinate system any point is considered as intersection of three mutually perpendicular surfaces.

- Circular cylinder (r = constant)
- Half plane with its edge along the z axis (ϕ = const.)
- Another plane xy (z = constant)

RELATION BETWEEN VARIABLES

- The relationships between the variables (x,y,z) of the Cartesian coordinate system and the cylindrical system (r,ϕ,z) are obtained as

$$r = \sqrt{x^2 + y^2} \qquad x = r\cos\phi$$

$$\phi = \tan^{-1} y/x \qquad y = r\sin\phi$$

$$z = z \qquad z = z$$

- So a point P (3, 4, 5) in Cartesian coordinate is the same as?

$$\rho = \sqrt{3^2 + 4^2} = 5$$

$$\phi = \tan^{-1} 4/3 = 0.927\,rad$$

$$z = 5$$

- So a point P (3, 4, 5) in Cartesian coordinate is the same as P (5, 0.927,5) in cylindrical coordinate)

SPHERICAL COORDINATES

- The spherical coordinate system is used dealing with problems having a degree of spherical symmetry.

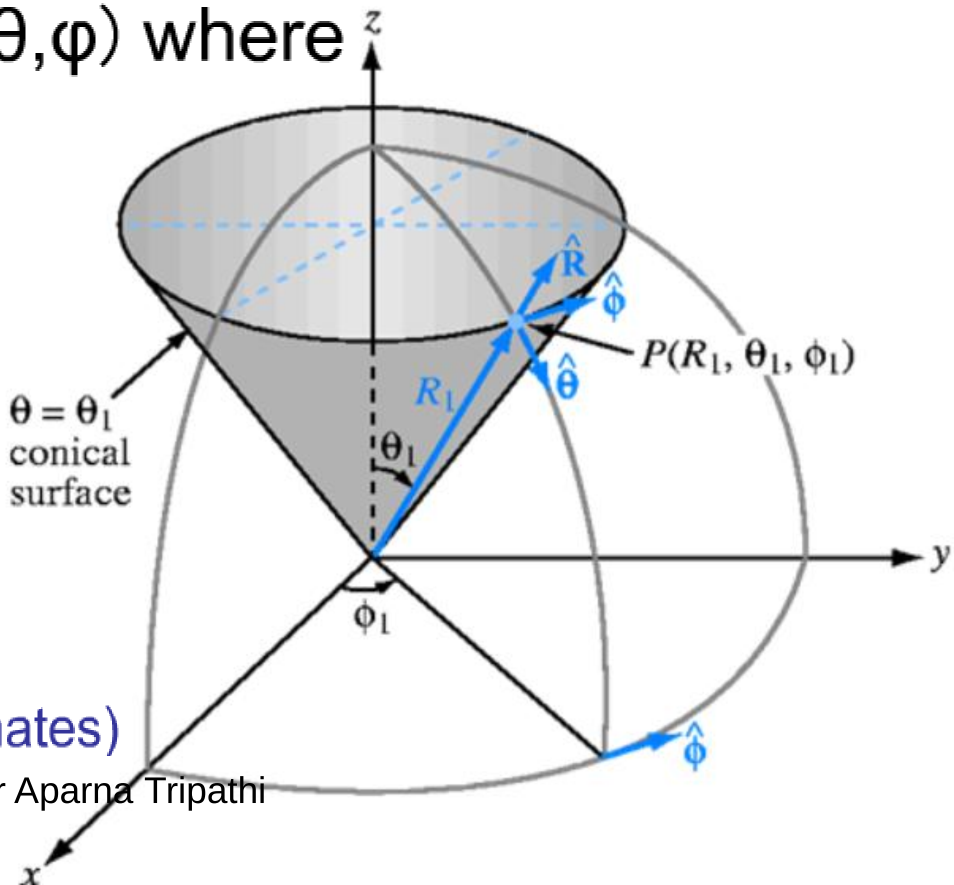
•The three mutually orthogonal surfaces are a sphere , a cone and a plane

- Point P represented as (r,θ,φ) where

r : the distance from the origin,

θ : called the *colatitude* is the angle between z-axis and vector of P,

Φ : *azimuthal* angle or the angular displacement from x-axis (the same azimuthal angle in cylindrical coordinates)



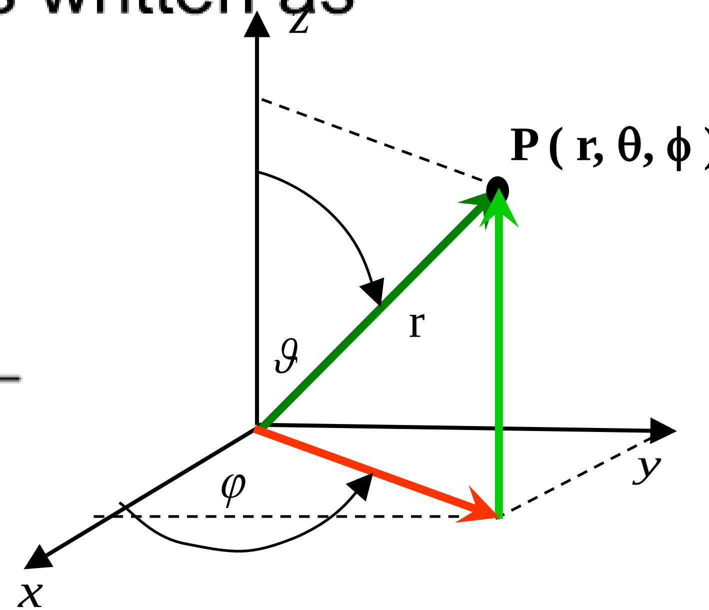
SPHERICAL COORDINATES

- The range of the variables are
  $0 \le r < \infty$, $0 \le \theta < \pi$, $0 < \varphi < 2\pi$

- A vector **A** in spherical coordinates written as
  $(A_r, A_\theta, A_\varphi)$ or $A_r a_r + A_\theta a_\theta + A_\varphi a_\varphi$

- The magnitude of A is

$$|\vec{A}| = \sqrt{A_r^{\ 2} + A_\phi^{\ 2} + A_\theta^{\ 2}}$$



Half plane with its edge along the z axis (ϕ = const.)
Sphere with its center at origin (r = constant)
Circular cone whose axis is the z axis and whose vertex is at the origin (θ= const.)

RELATION BETWEEN VARIABLES

$$r = \sqrt{x^2 + y^2 + z^2}$$

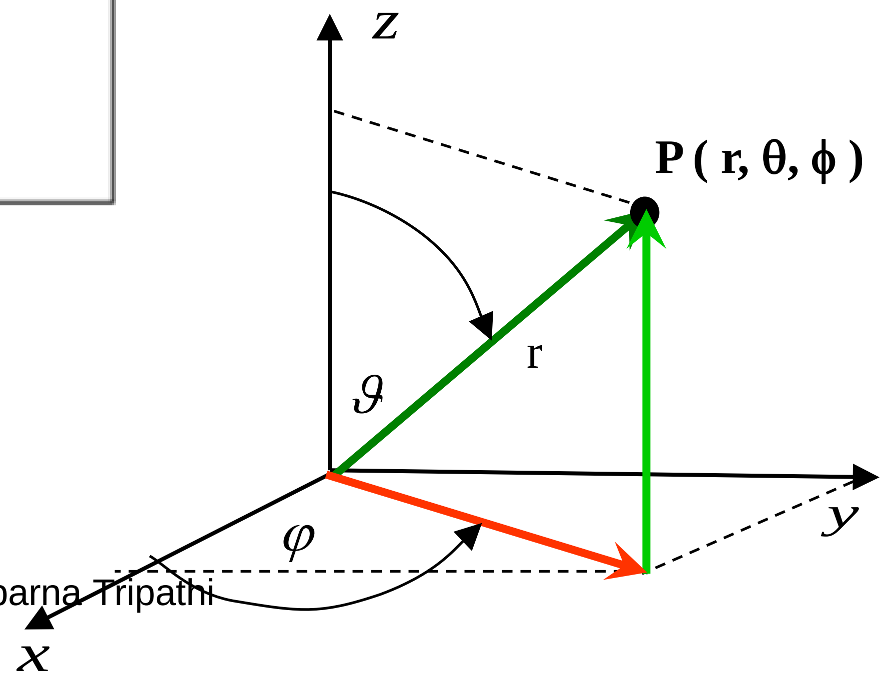
$$\theta = \tan^{-1} \frac{(\sqrt{x^2 + y^2})}{z}$$

$$\phi = \tan^{-1} \frac{y}{x}$$

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

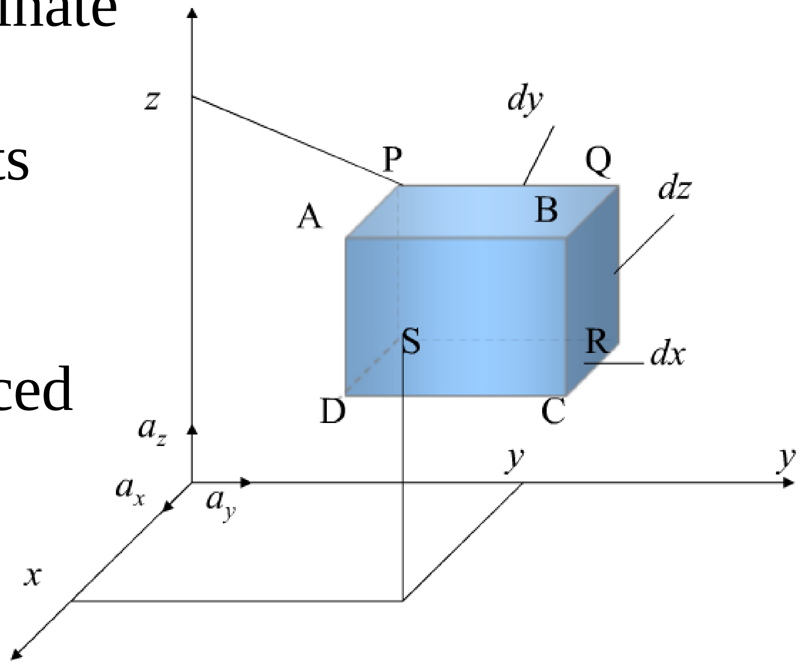
$$z = r \cos\theta$$



DIFFERENTIAL ELEMENTS

- In vector calculus the *differential elements* are length, area and volume are useful.
- They are defined in the Cartesian, cylindrical and spherical coordinate

CARTESIAN COORDINATES



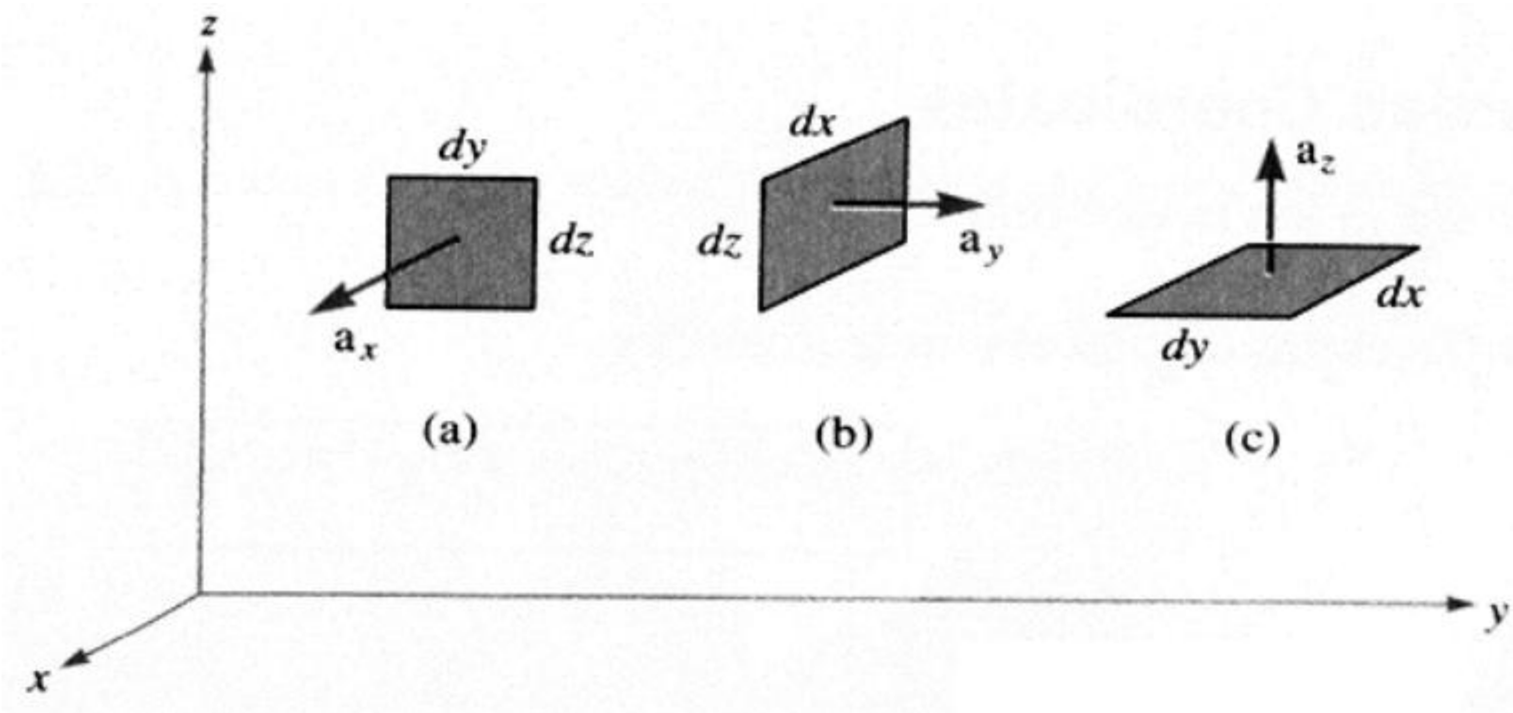
- Differential elements in Cartesian coordinate may be obtained by increasing coordinate values x,y, and z by differential increments dx,dy and dz.

- The this will lead to three slightly displaced planes intersecting at another pt P' (x+dx, y+dy and z+dz).

- Differential displacement dl is the distance between pt P and P'

Differential displacement : $d\vec{l} = dxa_x + dya_y + dza_z$

CARTESIAN COORDINATES



Differential normal area:

$$d\vec{S} = dydz\,\mathbf{a}_x$$

$$d\vec{S} = dxdz\,\mathbf{a}_y$$

$$d\vec{S} = dxdy\,\mathbf{a}_z$$

CARTESIAN COORDINATES

Differential displacement	$d\vec{l} = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z$
Differential normal area	$d\vec{S} = dydz\,\mathbf{a}_x$ $d\vec{S} = dxdz\,\mathbf{a}_y$ $d\vec{S} = dxdy\,\mathbf{a}_z$
Differential volume	$dv = dxdydz$

CYLINDRICAL COORDINATES

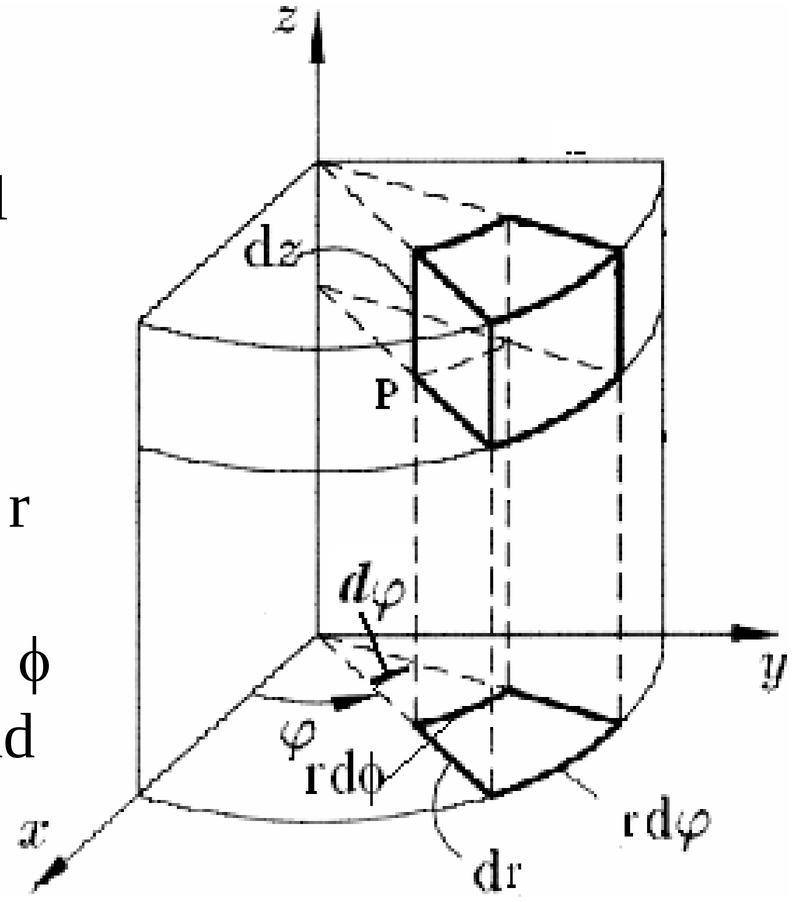
• Differential elements in Cylindrical coordinate may be obtained by increasing coordinate values r,ϕ, and z by differential increments dr,dϕ and dz.

•The this will lead to

➢two slightly displaced cylinders of radii r and r+dr,

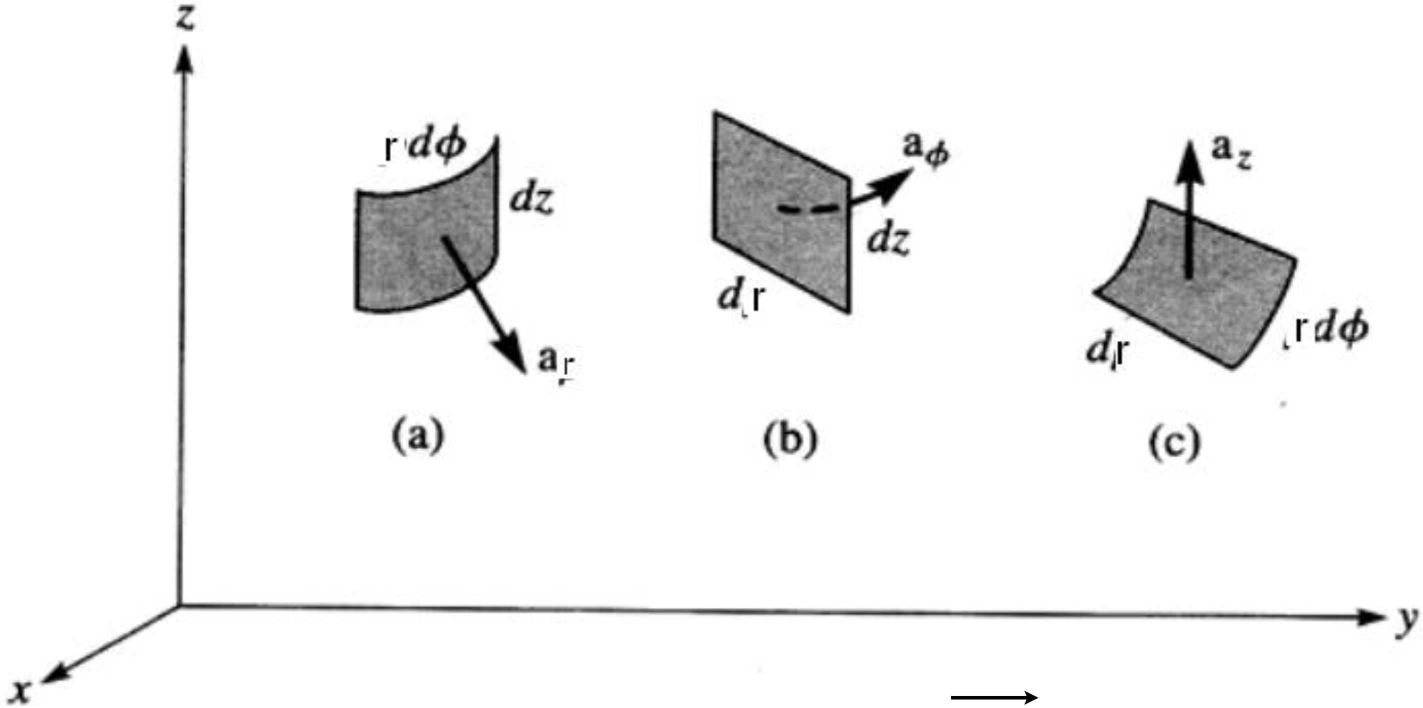
➢the two radial planes at angle ϕ and ϕ+d ϕ

➢The two horizontal planes at height z and z+dz



Differential displacement : $\overrightarrow{dl} = dr a_r + r d\phi a_\phi + dz a_z$

CYLINDRICAL COORDINATES



Differential normal area:

$$\overrightarrow{dS} = r d\phi dz a_r$$

$$\overrightarrow{dS} = dr dz a_\phi$$

$$\overrightarrow{dS} = r d\phi dr a_z$$

CYLINDRICAL COORDINATES

Differential displacement	$\overrightarrow{dl} = dr a_r + r d\phi a_\phi + dz a_z$
Differential normal area	$\overrightarrow{dS} = r d\phi dz a_r$ $\overrightarrow{dS} = dr dz a_\phi$ $\overrightarrow{dS} = r d\phi dr a_z$
Differential volume	$\overrightarrow{dv} = r\ dr\ d\varphi\ dz$

Example : 4

Use cylindrical coordinates to find the area of the curved surface of a right circular cylinder of radius a and height h

The differential surface element is dA = rdϕ dz

Then
$$A = \int_0^h \int_0^{2\pi} r d\phi dz$$

$$A = a\int_0^h \int_0^{2\pi} d\phi dz = 2\pi ah$$

Its volume (for a radius $r = a$) is
$$V = \int_0^a \int_0^h \int_0^{2\pi} r dr d\phi dz$$

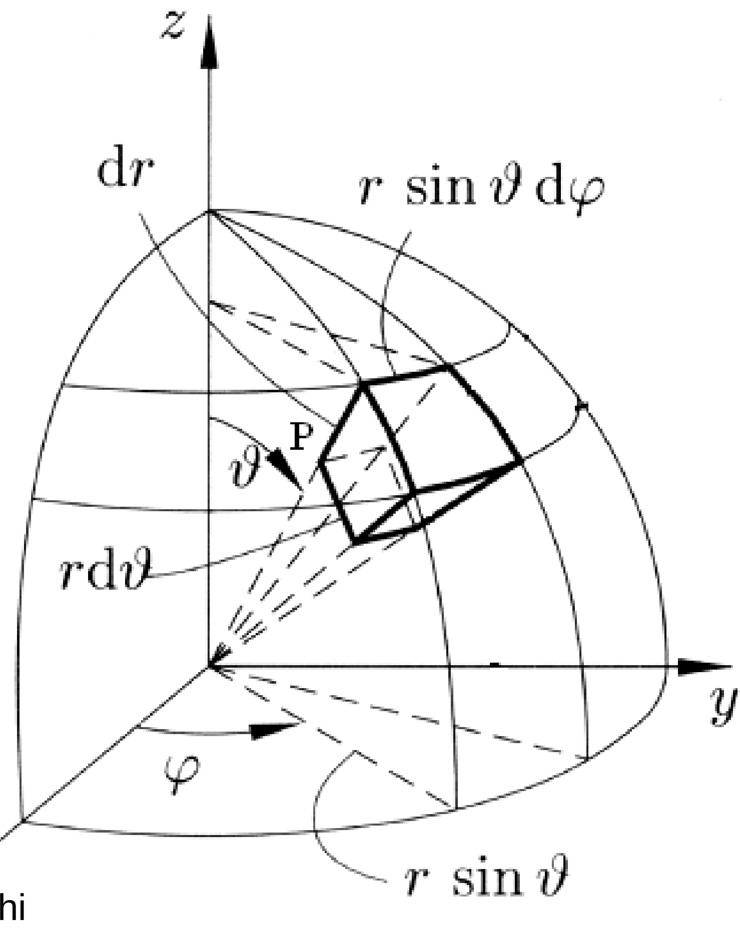
$$V = \pi a^2 h$$

SPHERICAL COORDINATES

• Differential elements in Spherical coordinate may be obtained by increasing coordinate values r,θ and ϕ by differential increments dr,d θ and dϕ.

•The this will lead to

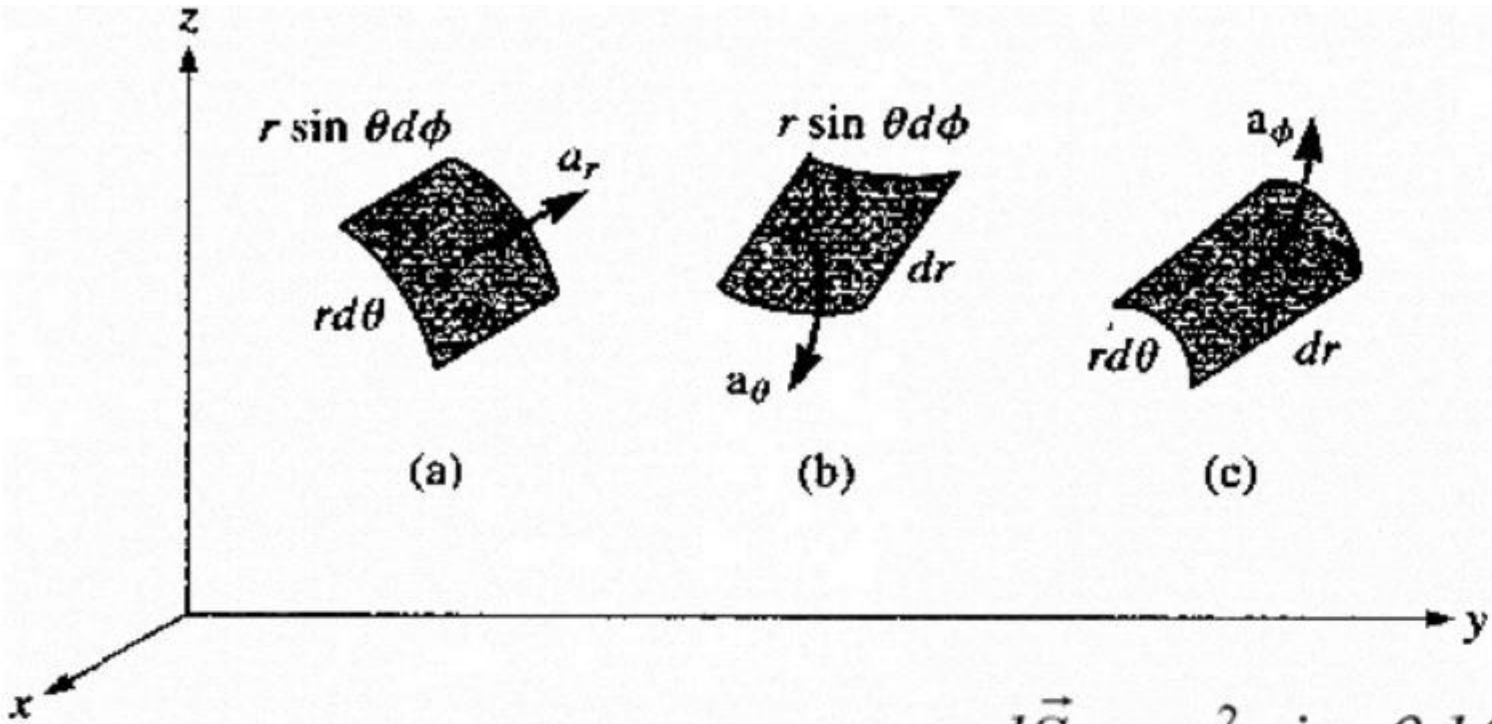
- Distance between two sphere - r and r+dr,
- Distance between two cone θ and θ +d θ
- Distance between two plane ϕ and ϕ+d ϕ



Differential line elements

$$dl^2 = dr^2 + (r\,d\theta)^2 + (r\sin\theta\,d\phi)^2$$

SPHERICAL COORDINATES



Differential normal area:

$$d\vec{S} = r^2 \sin\theta d\theta d\phi \mathbf{a}_r$$

$$d\vec{S} = r \sin\theta dr d\phi \mathbf{a}_\theta$$

$$d\vec{S} = r dr d\theta \mathbf{a}_\phi$$

SPHERICAL COORDINATES

Differential displacement	$d\vec{l} = dra_r + rd\theta a_\theta + r\sin\theta d\phi a_\phi$
Differential normal area	$d\vec{S} = r^2 \sin\theta d\theta d\phi a_r$ $d\vec{S} = r\sin\theta dr d\phi a_\theta$ $d\vec{S} = rdrd\theta a_\phi$
Differential volume	$dv = r^2 \sin\theta dr d\theta d\phi$