Lecture 11

Electromagnetic waves in Dielectric medium

Electromagnetic waves in Dielectric medium

- In an isotropic dielectric medium, there are no fundamental charge carriers, hence the current density in a dielectric medium is zero (J=0).

- There is no volume distribution od charge in the medium i.e. volume charge density is zero ($\boldsymbol{\rho} = 0$).

- Maxwell's eqs reduces to

$$\nabla \cdot \vec{E} = 0 \qquad .....(1) \qquad \mathrm{D} = \varepsilon \mathrm{E}$$

$$\nabla \cdot \vec{H} = 0 \qquad .....(2) \qquad \mathrm{B} = \mu \mathrm{H}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial H}{\partial t} \qquad ....(3)$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial E}{\partial t} \qquad ....(4)$$

Taking the curl of eq 3 both sides

$$\nabla \times \left(\nabla \times \vec{E}\right) = \nabla \times \left(-\mu \frac{\partial \overrightarrow{H}}{\partial t}\right) = -\mu \frac{\partial}{\partial t}\left(\nabla \times \overrightarrow{H}\right)$$

Now from vector identity

$$\nabla \times \left(\nabla \times \vec{E}\right) = \nabla\left(\nabla \bullet \vec{E}\right) - \left(\left(\nabla \bullet \nabla\right)\vec{E}\right) = \nabla\left(\nabla \bullet \vec{E}\right) - \nabla^2 \vec{E}$$

from Maxwell,s eq 4

$$\nabla \times \vec{H} = \varepsilon \frac{\partial E}{\partial t}$$

$$\nabla\left(\nabla \bullet \vec{E}\right) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t}\left(\varepsilon \frac{\partial E}{\partial t}\right)$$

from Maxwell,s eq 1

$$\nabla \bullet \vec{E} = 0$$

$$\nabla^2 \vec{E} = \mu \frac{\partial}{\partial t}\left(\varepsilon \frac{\partial E}{\partial t}\right) = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0$$

$$\nabla^2 \vec{E} - \frac{1}{v^2} \frac{\partial^2 E}{\partial t^2} = 0 \qquad \text{where v} = \frac{1}{\sqrt{\mu\varepsilon}} \text{ ....5}$$

Similarly Taking the curl of eq 4 both sides

$$\nabla \times \left(\nabla \times \overrightarrow{H}\right) = \nabla \times \left(\varepsilon \frac{\partial \overrightarrow{E}}{\partial t}\right) = \varepsilon \frac{\partial}{\partial t} \left(\nabla \times \vec{E}\right)$$

But from Maxwell, s eq 3 and eq2

$$\nabla \times \vec{E} = -\mu \frac{\partial H}{\partial t} \quad \text{and } \nabla \bullet H = 0$$

$$\nabla\left(\nabla \bullet H\right) - \nabla^2 H = \varepsilon \frac{\partial}{\partial t} \left(-\mu \frac{\partial H}{\partial t}\right)$$

$$0-\nabla^2 H = \varepsilon\,\frac{\partial}{\partial t}\left(-\mu\,\frac{\partial H}{\partial t}\right)$$

$$\nabla^2 H = \mu\varepsilon\,\frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 H - \mu\varepsilon\,\frac{\partial^2 H}{\partial t^2} = 0$$

$$\nabla^2 H - \frac{1}{v^2}\frac{\partial^2 H}{\partial t^2} = 0 \qquad \text{where v} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad ...6$$

Expression 5 and 6 are the wave equations for the propagation of electromagnetic waves in dielectric medium with a speed v .

$$\text{v} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{1}{\sqrt{\mu_r\varepsilon_r}}\times\frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} \qquad ...7$$

Where μ_r is the relative permeability and ε_r is the relative permittivity of the medium

$$\text{since} \quad \text{c} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$as\ \mu_r \rangle 1 \ \text{ and } \varepsilon_r \ \rangle 1$$

$$v \langle c$$

Thus, the velocity of propagation of a wave in a dielectric medium is less that that in air or free space.

The refractive index of the dielectric medium is defined as

$$\text{n} = \frac{\text{speed of wave in vacuum}}{\text{speed of wave in medium}} = \frac{c}{v} = \sqrt{\mu_r \varepsilon_r} \qquad \text{...8}$$

If the medium is non-magnetic then μ_r =1

$$\text{n} = \sqrt{\varepsilon_r} \quad \text{or} \quad \text{n}^2 = \varepsilon_r$$

Thus the refractive index of a non-magnetic dielectric medium is equal to the square root of its relative permittivity.

Solution of Electromagnetic waves for Dielectric medium

Assume we have a plane wave propagating in x direction (ie E, B not functions of y or z)

$$\nabla^2 E - \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} = 0 \qquad \ldots 9$$

$$\nabla^2 H - \frac{1}{v^2}\frac{\partial^2 H}{\partial t^2} = 0 \qquad \ldots 10$$

The wave solution of above eq in well known form may be written as

$$\overrightarrow{E}(r,t) = \overrightarrow{E_0}\, \mathrm{e}^{i(\vec{k}.\vec{r}-\omega t)}$$

$$\overrightarrow{H}(r,t) = \overrightarrow{H_0}\, \mathrm{e}^{i(\vec{k}.\vec{r}-\omega t)}$$

Where E_0 and H_0 are complex amplitudes which are constant in space and time

but $\vec{k}$ is a wave propagation vector and defined as

$$\vec{k} = k\,\hat{n} = \frac{2\pi}{\lambda}\,\hat{n} = \frac{2\pi\upsilon}{v}\,\hat{n} = \frac{\omega}{v}\,\hat{n}$$

Where n is a unit vector along the direction of propagation of em waves

The solution of plane em wave eqs in dielectric medium represented by eqs 9 and 10 satisfy Maxwell’s equation 1 and 2 only when

$$\nabla \cdot \vec{E} = 0 \qquad \text{when} \qquad \vec{k} \cdot \vec{E} = 0$$

$$\text{similarly}$$

$$\nabla \cdot \overrightarrow{H} = 0 \qquad \text{when} \qquad \vec{k} \cdot \overrightarrow{H} = 0$$

This indicates that electric and magnetic fields are ⊥ to the direction of propagation vector k i.e the em waves in isotropic dielectric are transverse in nature.

$$\text{Maxwell's Eq 3 and eq4 in isotropic dielectric}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \overrightarrow{H}}{\partial t}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \overrightarrow{E}}{\partial t}$$

$$\vec{k} \times \vec{E} = \mu\omega\overrightarrow{H} \qquad \ldots 11$$

Similarly for $\vec{k} \times \overrightarrow{H}$ may be obtained as

$$\vec{k} \times \overrightarrow{H} = -\varepsilon\omega\vec{E} \qquad \ldots 12$$

From eq 11, the field vector H is ⊥ to both k and E and according to eq 12, E is ⊥ to both k and H.

So its concluded that field vector E and H are mutually ⊥ to each other and also ⊥ to the direction of propagation of wave.

from eq 11

$$\mu\omega\overrightarrow{H} = \vec{k} \times \vec{E} \qquad \ldots\text{A} \qquad (\vec{k} = k\,\hat{n})$$

$$\mu\omega\overrightarrow{H} = k\left(\hat{n} \times \vec{E}\right)$$

Where n is a unit vector along the direction of propagation of em waves

$$\overrightarrow{H} = \frac{k}{\mu\omega}\left(\hat{n} \times \vec{E}\right)$$

but $\vec{k}$ is a wave propagation vector and defined as

$$\vec{k} = k\,\hat{n} = \frac{2\pi}{\lambda}\hat{n} = \frac{2\pi\upsilon}{v}\hat{n} = \frac{\omega}{v}\hat{n}$$

$$\therefore \quad \overrightarrow{H} = \frac{1}{\mu v}\left(\hat{n} \times \vec{E}\right)$$

in terms of magnitude

$$\left|\overrightarrow{H}\right| = \sqrt{\frac{\varepsilon}{\mu}}\left|\hat{n} \times \vec{E}\right| \qquad \because k = \frac{\omega}{v} \quad \text{and } v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$H = \sqrt{\frac{\varepsilon}{\mu}} E$$

Now the ratio of magnitude of E to the magnitude of H is symbolized by Z

$$Z = \frac{E}{H} = \frac{E_0}{H_0} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$$

This relation shows that the field vector E and H are in same phase.
Z is called the wave impedance of isotropic dielectric medium.

The wave impedance of medium is related to that of free space by the relation

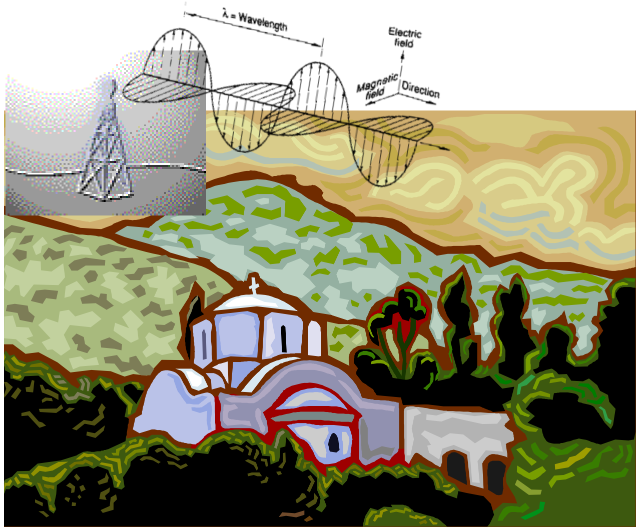
$$Z = \sqrt{\frac{\mu_r}{\varepsilon_r} \cdot \frac{\mu_0}{\varepsilon_0}} = Z_0 \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

where $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ is called the wave impedance

in free space.

Power in a wave

- A wave **carries power** and **transmits** it wherever it goes



The rate of flow of energy per unit area in wave is given by the *Poynting vector.*

Poynting Vector Derivation

Taking scalar product of Maxwell ‘s eq 3 with H and eq 4 with E

$$H\cdot\nabla\times E=-H\cdot\frac{\partial B}{\partial t}\qquad ...1$$

$$\mathrm{E}\cdot(\nabla\times\mathrm{H})=\mathrm{E}\cdot\mathrm{J}+\mathrm{E}\cdot\frac{\partial D}{\partial t}\qquad ...2$$

Subtracting eq 2 from eq 1

$$H\cdot\nabla\times E-\mathrm{E}\cdot(\nabla\times\mathrm{H})=-H\cdot\frac{\partial B}{\partial t}-\ \mathrm{E}\cdot\mathrm{J}-\mathrm{E}\cdot\frac{\partial D}{\partial t}$$

$$H\cdot\nabla\times E-\mathrm{E}\cdot(\nabla\times\mathrm{H})=-\left[H\cdot\frac{\partial B}{\partial t}+\mathrm{E}\cdot\frac{\partial D}{\partial t}\right]-\ \mathrm{E}\cdot\mathrm{J}\qquad ...3$$

Now using vector identity

$$\nabla\cdot(A\times B)=B\cdot(\nabla\times A)-A\cdot(\nabla\times B)\text{or in this case :}$$

$$\nabla\cdot(E\times H)=H\cdot(\nabla\times E)-E\cdot(\nabla\times H)$$

Eq 3 can be written as

$$\nabla \cdot \left(E \times \mathrm{H}\right) = -\left[H \cdot \frac{\partial B}{\partial t} + \mathrm{E} \cdot \frac{\partial D}{\partial t}\right] - \ \mathrm{E} \cdot \mathrm{J} \qquad ...4$$

Using relation B=μH and D=εE, in eq 4

$$\nabla \cdot \left(E \times \mathrm{H}\right) = -\left[H \cdot \frac{\partial\left(\mu H\right)}{\partial t} + \mathrm{E} \cdot \frac{\partial\left(\varepsilon E\right)}{\partial t}\right] - \ \mathrm{E} \cdot \mathrm{J}$$

$$= -\mu H \cdot \frac{\partial H}{\partial t} - \varepsilon \mathrm{E} \cdot \frac{\partial E}{\partial t} - \ \mathrm{E} \cdot \mathrm{J} \qquad ....5$$

But

$$H \cdot \frac{\partial H}{\partial t} = \frac{1}{2}\frac{\partial\left(H\right)^2}{\partial t} \qquad \text{and} \qquad E \cdot \frac{\partial E}{\partial t} = \frac{1}{2}\frac{\partial\left(E\right)^2}{\partial t}$$

$$\nabla \cdot \left(E \times \mathrm{H}\right) = -\frac{\mu}{2}\frac{\partial\left(H\right)^2}{\partial t} - \frac{\varepsilon}{2}\frac{\partial\left(E\right)^2}{\partial t} - \ \mathrm{E} \cdot \mathrm{J}$$

$$\nabla\cdot\left(E\times \mathrm{H}\right)=\frac{\partial}{\partial t}\left[\frac{\mu H^{2}}{2}+\frac{\varepsilon E^{2}}{2}\right]-\ \ \mathrm{E}\cdot\mathrm{J}\quad \ldots 6$$

Now integrating eq 6 over a volume V bounded by surface S

$$\int_{v}\nabla\cdot\left(E\times H\right)dv=-\frac{\partial}{\partial t}\int_{v}\left(\frac{\varepsilon}{2}E^{2}+\frac{\mu}{2}H^{2}\right)dv-\int_{v}E\cdot J\ dv\quad \ldots 7$$

Using Gauss Divergence theorem $\int_{v}\nabla\cdot\left(E\times H\right)dv=\oint_{s}\left(E\times H\right)ds$

Eq 7 becomes

$$\oint_{S}\left(E\times H\right)\cdot dS=-\frac{\partial}{\partial t}\int_{v}\left(\frac{\varepsilon}{2}E^{2}+\frac{\mu}{2}H^{2}\right)dv-\int_{v}E\cdot J dv\quad \ldots 9$$

Which means that the total power coming out of a volume is either due to the electric or magnetic field energy variations or is lost in ohmic losses.

Poynting Vector

- Waves carry energy and information

- Poynting says that the net power flowing out of a given volume is = to the decrease in time in energy stored minus the conduction losses.

$$\vec{S} = \vec{E} \times \vec{H} \quad [\mathrm{W/m^2}]$$

Represents the instantaneous power density vector associated to the electromagnetic wave.

- The Poynting vector has the same direction as the direction of propagation.

The Poynting Vector

Energy transport is defined by the Poynting vector S as:

$$\boxed{\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_0}}$$

The direction of S is the direction of propagation of the wave

$$\boxed{S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{E^2}{Z_0} = \frac{E^2}{377\Omega}}$$

Electrostatic Boundary Conditions

- **At any point on the boundary,**
  - the components of ***E*$_1$** and ***E*$_2$** tangential to the boundary are equal

$$\boxed{E_{1t} = E_{2t}}$$

  - the components of ***D*$_1$** and ***D*$_2$** normal to the boundary are equal

$$\boxed{D_{1t} = D_{2t}}$$

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Magnetic Boundary Conditions

- The normal component of $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$ continuous across a interface:

$$\boxed{B_{1n} = B_{2n}}$$

- The tangential component of a $\boldsymbol{H_1}$ and $\boldsymbol{H_2}$ to the boundary are equal

$$\boxed{H_{1t} = H_{2t}}$$

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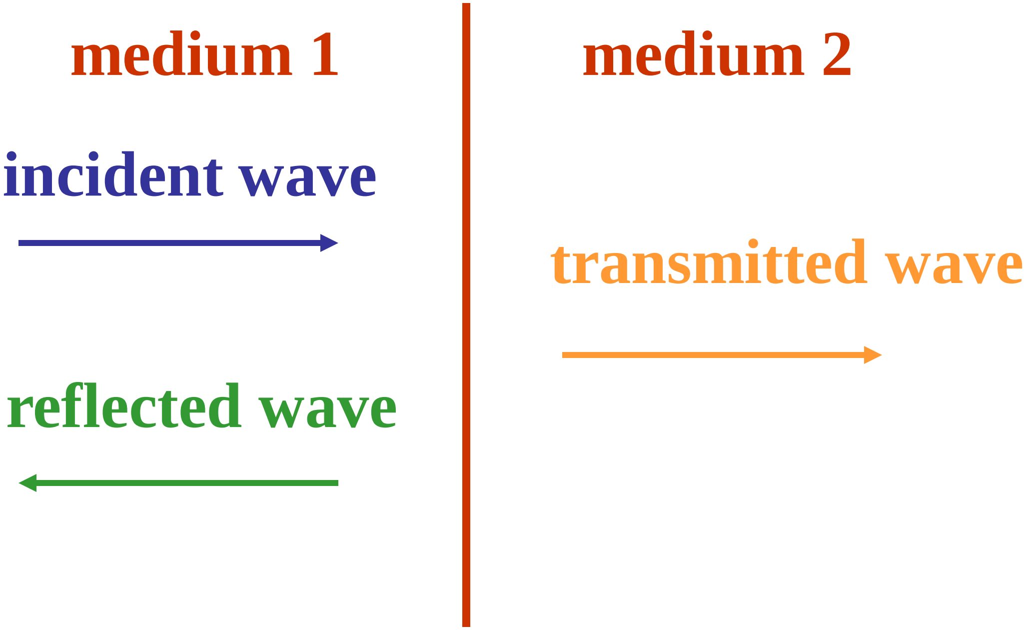
Reflection and Transmission of Waves at Planar Interfaces



medium 1

medium 2

incident wave

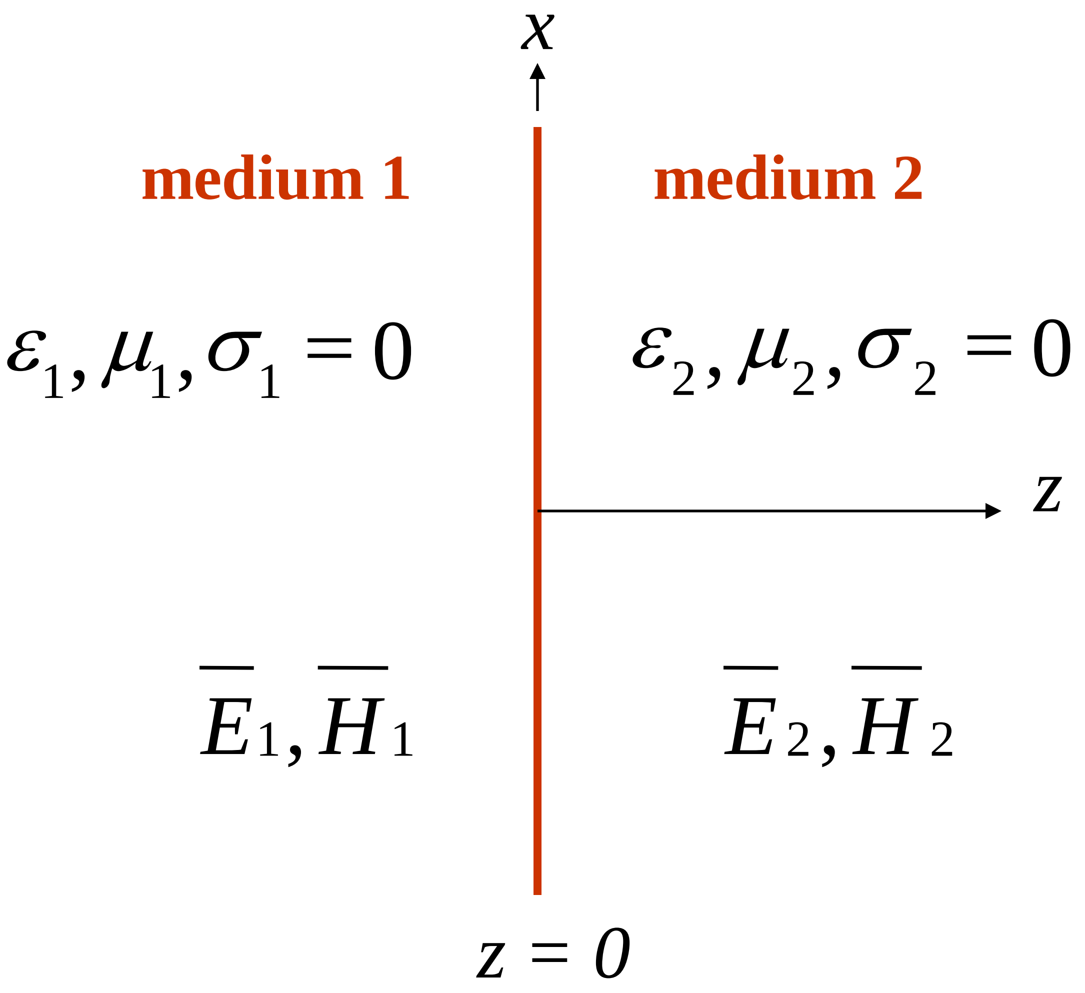
transmitted wave

reflected wave

Normal Incidence

- Consider both medium 1 and medium 2
- Let us place the boundary between the two media in the $z = 0$ plane, and consider an incident plane wave which is traveling in the $+z$-direction.
- we assume that the electric field of the incident wave is in the x-direction.

Normal Incidence



medium 1

medium 2

$\varepsilon_1, \mu_1, \sigma_1 = 0$

$\varepsilon_2, \mu_2, \sigma_2 = 0$

$\overline{E}_1, \overline{H}_1$

$\overline{E}_2, \overline{H}_2$

$z = 0$

Normal Incidence

- Incident wave

known

$$\overline{E}_i = \hat{a}_x E_{i0} e^{-j\beta_1 z}$$

$$\overline{H}_i = \frac{1}{\eta_1} \hat{a}_z \times \overline{E}_i = \hat{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z}$$

$$\beta_1 = \omega\sqrt{\varepsilon_1 \mu_1} \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

Normal Incidence

- Reflected wave

unknown

$$\overline{E}_r = \hat{a}_x E_{r0} e^{+j\beta_1 z}$$

$$\overline{H}_r = \frac{1}{\eta_1}\left(-\hat{a}_z\right)\times\overline{E}_r = -\hat{a}_y \frac{E_{r0}}{\eta_1} e^{+j\beta_1 z}$$

Normal Incidence

- Transmitted wave

unknown

$$\overline{E}_t = \hat{a}_x E_{t0} e^{-j\beta_2 z}$$

$$\overline{H}_t = \frac{1}{\eta_2} \hat{a}_z \times \overline{E}_t = \hat{a}_y \frac{E_{t0}}{\eta_2} e^{-j\beta_2 z}$$

$$\beta_2 = \omega \sqrt{\varepsilon_2 \mu_2} \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

Normal Incidence

- The total electric and magnetic fields in medium 1 are

$$\overline{E}_1 = \overline{E}_i + \overline{E}_r = \hat{a}_x \left[E_{i0} e^{-j\beta_1 z} + E_{r0} e^{+j\beta_1 z} \right]$$

$$\overline{H}_1 = \overline{H}_i + \overline{H}_r = \hat{a}_y \left[\frac{E_{i0}}{\eta_1} e^{-j\beta_1 z} - \frac{E_{r0}}{\eta_1} e^{+j\beta_1 z} \right]$$

Normal Incidence

- The total electric and magnetic fields in medium 2 are

$$\overline{E}_2 = \overline{E}_t = \hat{a}_x E_{t0} e^{-j\beta_2 z}$$

$$\overline{H}_2 = \overline{H}_t = \hat{a}_y \frac{E_{t0}}{\eta_2} e^{-j\beta_2 z}$$

Normal Incidence

- To determine the unknowns E_{r0} and E_{t0}, we must enforce the BCs at $z = 0$:

$$\overline{E}_1(z=0)=\overline{E}_2(z=0)$$

$$\overline{H}_1(z=0)=\overline{H}_2(z=0)$$

Normal Incidence)

- From the BCs we have

$$E_{i0} + E_{r0} = E_{t0}$$

or

$$\frac{E_{i0}}{\eta_1} - \frac{E_{r0}}{\eta_1} = \frac{E_{t0}}{\eta_2}$$

$$E_{r0} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{i0}, \qquad E_{t0} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{i0}$$

Reflection and Transmission Coefficients

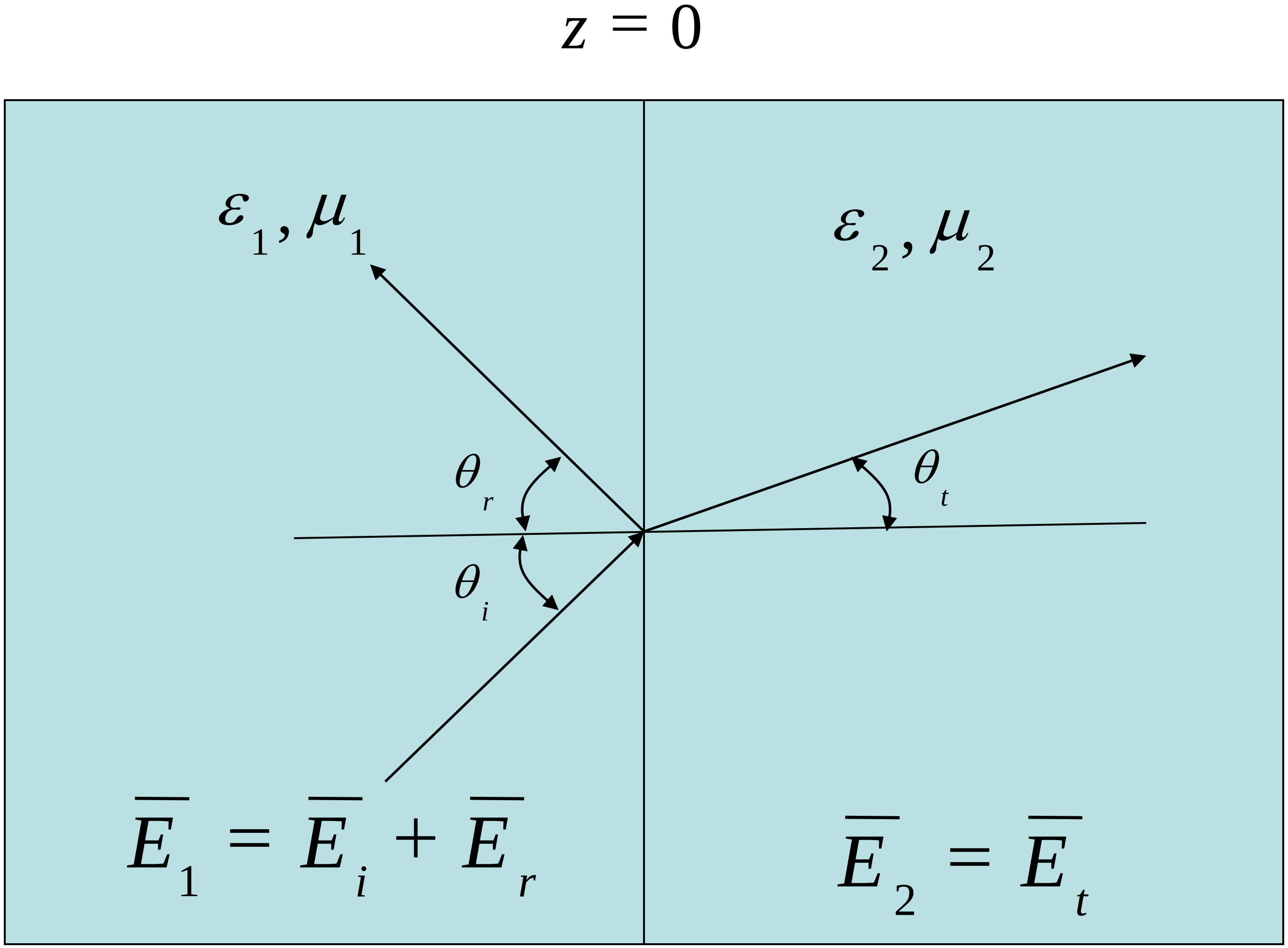
- Define the ***reflection coefficient*** as

$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

- Define the ***transmission coefficient*** as

$$\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

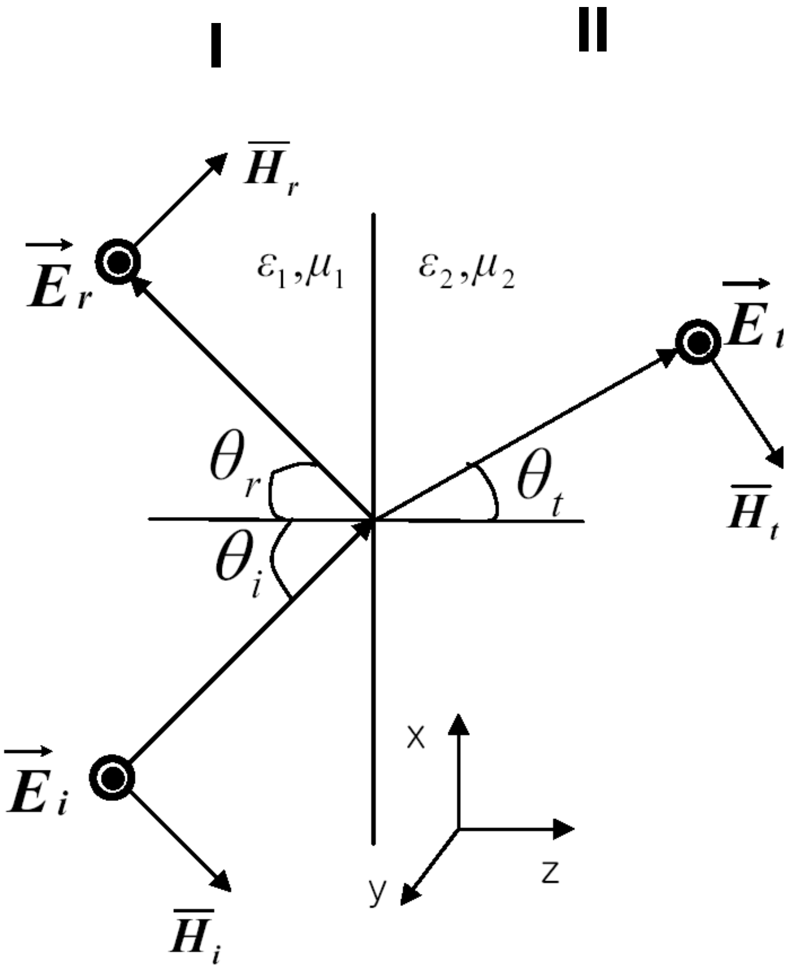
Oblique Incidence



$$\overline{E}_1 = \overline{E}_i + \overline{E}_r$$

$$\overline{E}_2 = \overline{E}_t$$

Perpendicular Polarization



$$\overline{E}_i = E_{i0}\exp(-j\beta_1 z)\,a_x$$

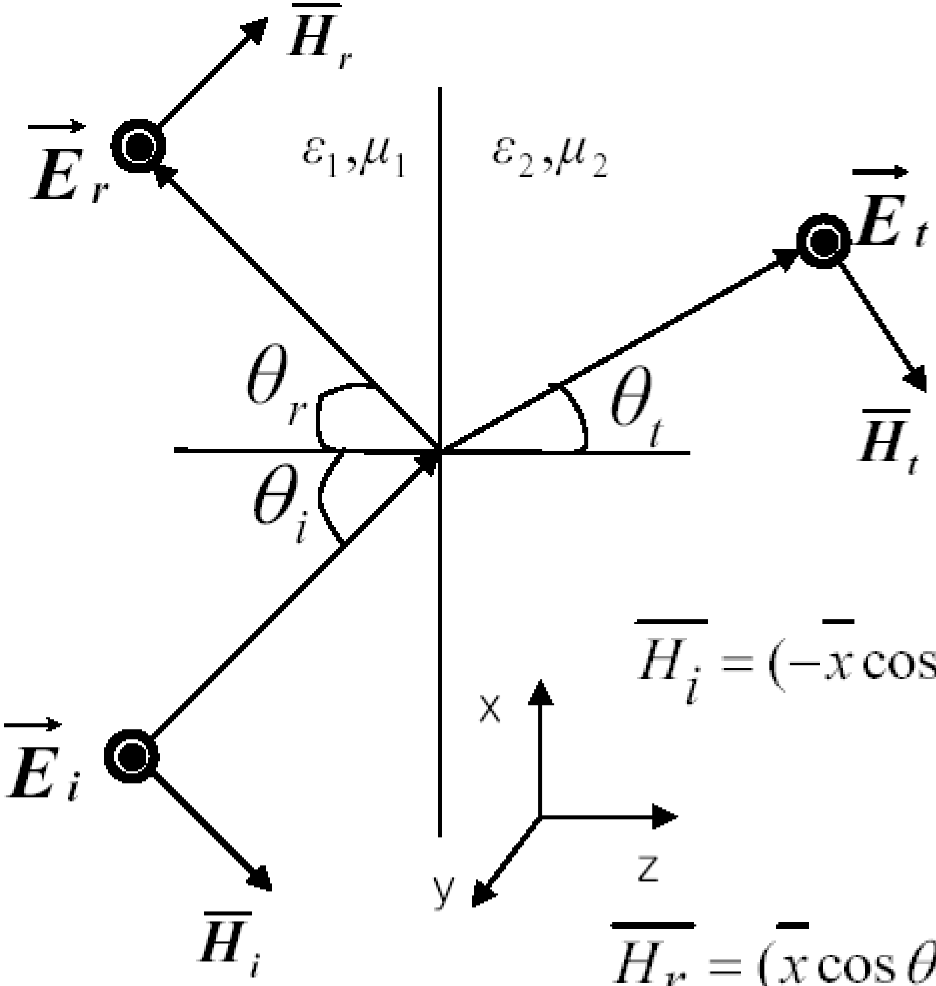
$$\overline{E_i} = \overline{y}E_i\exp(-j\beta_{1x}x)\exp(-j\beta_{1z}z)$$

$$\overline{E_r} = \overline{y}\,\Gamma E_i\exp(-j\beta_{rx}x)\exp(j\beta_{rz}z)$$

$$\overline{E_t} = \overline{y}\,\tau E_i\exp(-j\beta_{2x}x)\exp(-j\beta_{2z}z)$$

$$\Gamma = \frac{E_r}{E_i}$$

$$\tau = \frac{E_t}{E_i}$$



$$\overline{H_i} = (-\bar{x}\cos\theta_i + \bar{z}\sin\theta_i)\frac{E_i}{\eta_1}\exp(-j\beta_{1x}x)\exp(-j\beta_{1z}z)$$

$$\overline{H_r} = (\bar{x}\cos\theta_r + \bar{z}\sin\theta_r)\frac{\Gamma E_i}{\eta_1}\exp(-j\beta_{rx}x)\exp(j\beta_{rz}z)$$

$$\overline{H_t} = (-\bar{x}\cos\theta_t + \bar{z}\sin\theta_t)\frac{\tau E_i}{\eta_2}\exp(-j\beta_{2x}x)\exp(-j\beta_{2z}z)$$

B.C.'s at $z=0$

1) $\overline{E}_{tan}$ coutinuous $(\overline{E_i}+\overline{E_r}=\overline{E_t})$

$$\overline{E_i} = \overline{y} E_i \exp(-j\beta_{1x}x)\exp(-j\beta_{1z}z)$$

$$\overline{E_r} = \overline{y}\ \Gamma E_i \exp(-j\beta_{rx}x)\exp(j\beta_{rz}z)$$

$$\overline{E_t} = \overline{y}\ \tau E_i \exp(-j\beta_{2x}x)\exp(-j\beta_{2z}z)$$

$$\exp(-j\beta_{1x}x) + \Gamma\exp(-j\beta_{rx}x) = \tau\exp(-j\beta_{2x}x) \qquad \text{……(A)}$$

$$\beta_{1x} = \beta_{rx} = \beta_{2x}$$

$$1+\Gamma=\tau \qquad \text{…..(B)}$$

B.C.'s at z=0

2) $\bar{H}_{\tan}$ continuous $(\bar{H}_i + \bar{H}_r = \bar{H}_t)$

$$-\frac{\cos\theta_i}{\eta_1} + \frac{\cos\theta_i}{\eta_1}\Gamma = -\frac{\cos\theta_t}{\eta_2}\tau$$ **……(C)**

With $1+\Gamma=\tau$,

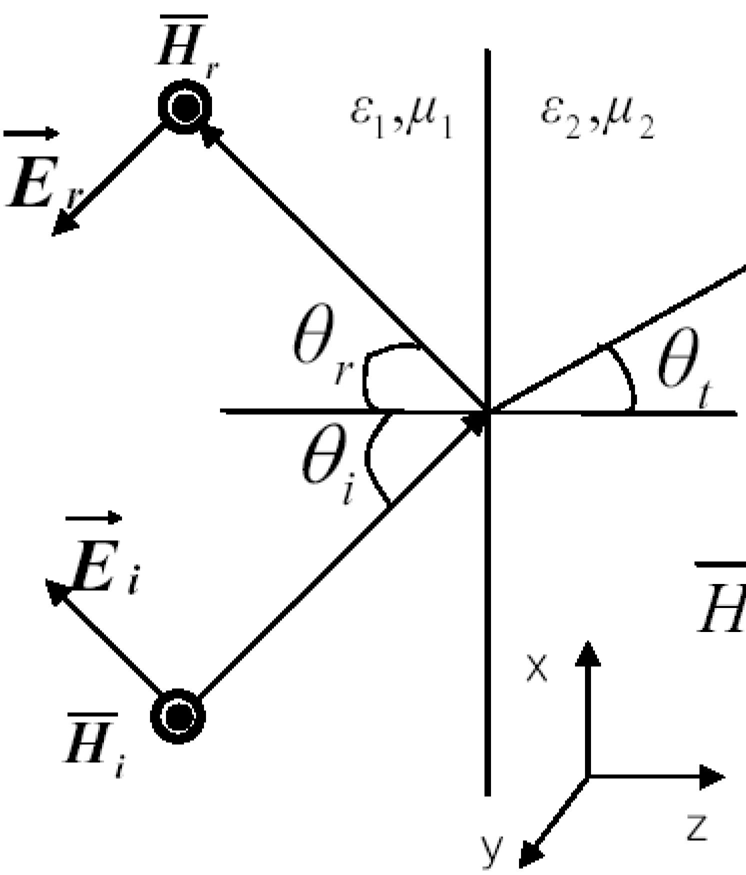
reflection coefficient

$$\Gamma_\perp = \frac{\eta_2/\cos\theta_t - \eta_1/\cos\theta_i}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i}$$

transmission coefficient

$$\tau_\perp = \frac{2\eta_2/\cos\theta_t}{\eta_2/\cos\theta_t + \eta_1/\cos\theta_i}$$

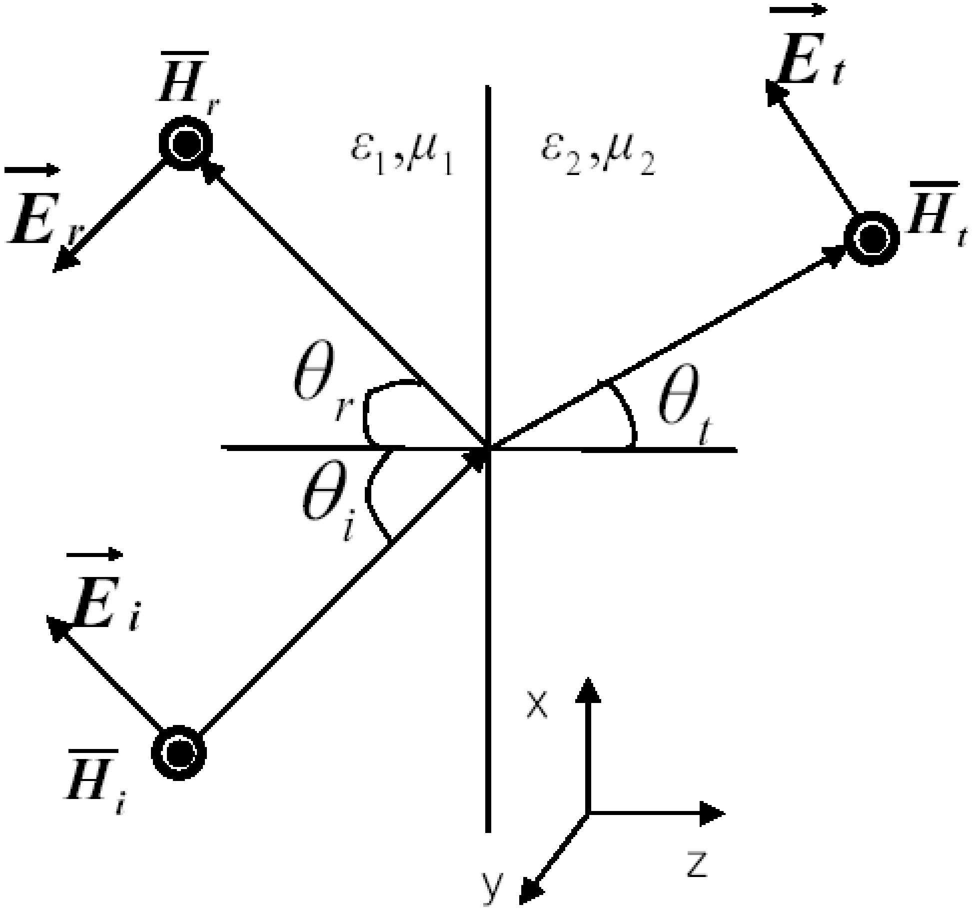
Parallel Polarization



$$\overline{H_i} = \overline{y} H_i \exp(-j\beta_{1x} x) \exp(-j\beta_{1z} z)$$

$$\overline{H_r} = \overline{y} H_r \exp(-j\beta_{rx} x) \exp(j\beta_{rz} z)$$

$$\overline{H_t} = \overline{y} H_t \exp(-j\beta_{2x} x) \exp(-j\beta_{2z} z)$$



$$E = H\eta$$

$$\overline{E_i} = (\overline{x}\cos\theta_i - \overline{z}\sin\theta_i)H_i\eta_1 \exp(-j\beta_{1x}x)\exp(-j\beta_{1z}z)$$

$$\overline{E_r} = (-\overline{x}\cos\theta_r - \overline{z}\sin\theta_r)H_r\eta_1 \exp(-j\beta_{rx}x)\exp(j\beta_{rz}z)$$

$$\overline{E_t} = (\overline{x}\cos\theta_t - \overline{z}\sin\theta_t)H_t\eta_2 \exp(-j\beta_{2x}x)\exp(-j\beta_{2z}z)$$

$$\frac{H_r}{H_i} = -\Gamma \qquad \frac{H_t}{H_i} = \frac{E_t / \eta_2}{E_i /} = \tau \frac{\eta_1}{\eta_2}$$

B.C.'s at $z=0$

1) $\bar{H}_{\tan}$ coutinuous $(\bar{H}_i + \bar{H}_r = \bar{H}_t)$

$$\overline{H_i} = \bar{y} H_i \exp(-j\beta_{1x}x)\exp(-j\beta_{1z}z)$$

$$\overline{H_r} = \bar{y} H_r \exp(-j\beta_{rx}x)\exp(j\beta_{rz}z)$$

$$\overline{H_t} = \bar{y} H_t \exp(-j\beta_{2x}x)\exp(-j\beta_{2z}z)$$

$$\exp(-j\beta_{1x}x) - \Gamma\exp(-j\beta_{rx}x) = \frac{\eta_1}{\eta_2}\tau\exp(-j\beta_{2x}x)$$

$$\beta_{1x} = \beta_{rx} = \beta_{2x}$$

$$1 - \Gamma = \frac{\eta_1}{\eta_2}\tau$$ …..(A)

B.C.'s at $z = 0$

2) $\overline{E}_{\tan}$ continuous

$$\cos\theta_i + \Gamma\cos\theta_i = \tau\cos\theta_t$$ ……..(B)

reflection coefficient

$$\boxed{\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}}$$

transmission coefficient

$$\boxed{\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}}$$