Physics-II (10B11PH211)

BY

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Outline of the Course

1. **Electromagnetic Theory**
2. **Thermodynamics**
3. **Quantum Mechanics**
4. **Solid State Physics or Condensed Matter Physics**

Electromagnetic Theory

- Fundamentals of Vector Calculus which include, fields, gradient, divergence and curl
- Coulomb’s law, electric flux and, Gauss’s law, its proof for the charge inside and outside the Gaussian surface, applications of Gauss law for spherically and cylindrically symmetric problems
- Electric field due to charged conductor, force per unit area on the surface of the charged conductor, treatment of electrostatic problems by solution of laplace and poisson’s equations
- Biot-Savart law, Ampere’s law, Maxwell’s equations (derivations) in free space and dielectric media

Electromagnetic Theory continued · · ·

- Plane electromagnetic waves in free space, transverse nature, wave impedance and energy flow
- Energy in electromagnetic waves (Poynting vector and Poynting theorem)
- Derivations of expressions for energy density and energy flux (Poynting vector) in an electromagnetic field, radiation pressure, boundary conditions across the medium (without proof)
- Propagation of EM waves (i.e., light) through boundary- reflection, refraction, absorption (normal incidence), oblique incidence and total internal reflection.

Thermodynamics

- Review of thermodynamical systems and processes, zeroth law of thermodynamics, first law of thermodynamics
- Specific heat relation, isothermal, adiabatic, isochoric and isobaric processes, gas equation during an adiabatic process, slopes of adiabatic and isothermals
- Work done during an isothermal and adiabatic process, relation between adiabatic and isothermal elasticities, second law of thermodynamics, concept of entropy, calculation of entropy for an ideal gas (pressure volume and temperature), principle of increase of entropy or degradation of energy
- Reversible and irreversible processes, Carnot cycle and Carnot engine, refrigerator, rankine cycle (Steam engine), Otto cycle(Petrol engine), diesel engine, phase transitions, Clausius-Cleyperon equation
- Thermodynamic Potentials(Internal energy, Enthalpy, Helmholtz free energy, Gibb’s free energy, Maxwell’s equations

Quantum Mechanics

- Wave particle duality, de-Broglie concept of matter waves, wavelength expression for different cases, Davisson & Germer experiment, G.P. Thomson experiment, interpretation of Bohr's quantization rule, concept of wave packet
- Phase and group velocities and their derivations for a matter wave, Heisenberg uncertainty principle. experimental illustration (Position of a particle by high power Microscope, Diffraction of electron beam by a single slit), applications of uncertainty principle (Non existence of electron in the nucleus)
- Radius of the Bohr's first orbit, zero point energy of harmonic oscillator, finite width of spectral lines

Quantum Mechanics continued · · ·

- Time-independent and time-dependent Schrödinger wave equation, physical significance of wave function.
- Normalized and orthogonal wave functions, operators and their representation, expectation value
- Particle in one dimensional box, extension to 3-dimensional box, potential barrier and harmonic oscillator

Solid State Physics

- Lattice points and space lattice, Basis and crystal structure, unit cell and primitive cell, seven crystal systems and fourteen Bravais space lattice, coordination number, nearest neighbor distance, atomic radius, atomic packing factor in crystal structure, calculation of lattice constant, lattice planes and Miller indices
- Separation between lattice planes, derivation and examples, X-ray diffraction, Bragg's law of X-ray diffraction, Bragg's X-ray spectrometer, powder crystal method, rotating crystal method, basic ideas of bonding
- Bonding in solids

Solid State Physics continued · · ·

- Electronic conduction in metals, classical free electron theory, quantum theory of free electrons, band theory of solids, Kronig-Penny model and its interpretation
- Brillouin zones, distinction between metals, semiconductors and insulators, intrinsic and extrinsic semiconductors
- Carrier concentration in thermal equilibrium in intrinsic semiconductor, Fermi level and energy band diagram in intrinsic semiconductor, energy band diagram and Fermi level in extrinsic semiconductors, effect of temperature on extrinsic semiconductor
- Electrical conductivity of intrinsic semiconductor and extrinsic semiconductor, Hall effect, allied parameters and its applications

Books Recommended

Electromagnetic Theory

- Introduction to Electrodynamics By: David J. Griffiths
- Schaum's Outline of Theory and Problems of Electromagnetics
- Classical Electrodynamics By: J.D. Jackson

Thermodynamics

- Heat and Thermodynamics: Mark Waldo Zemansky, Richard Dittman

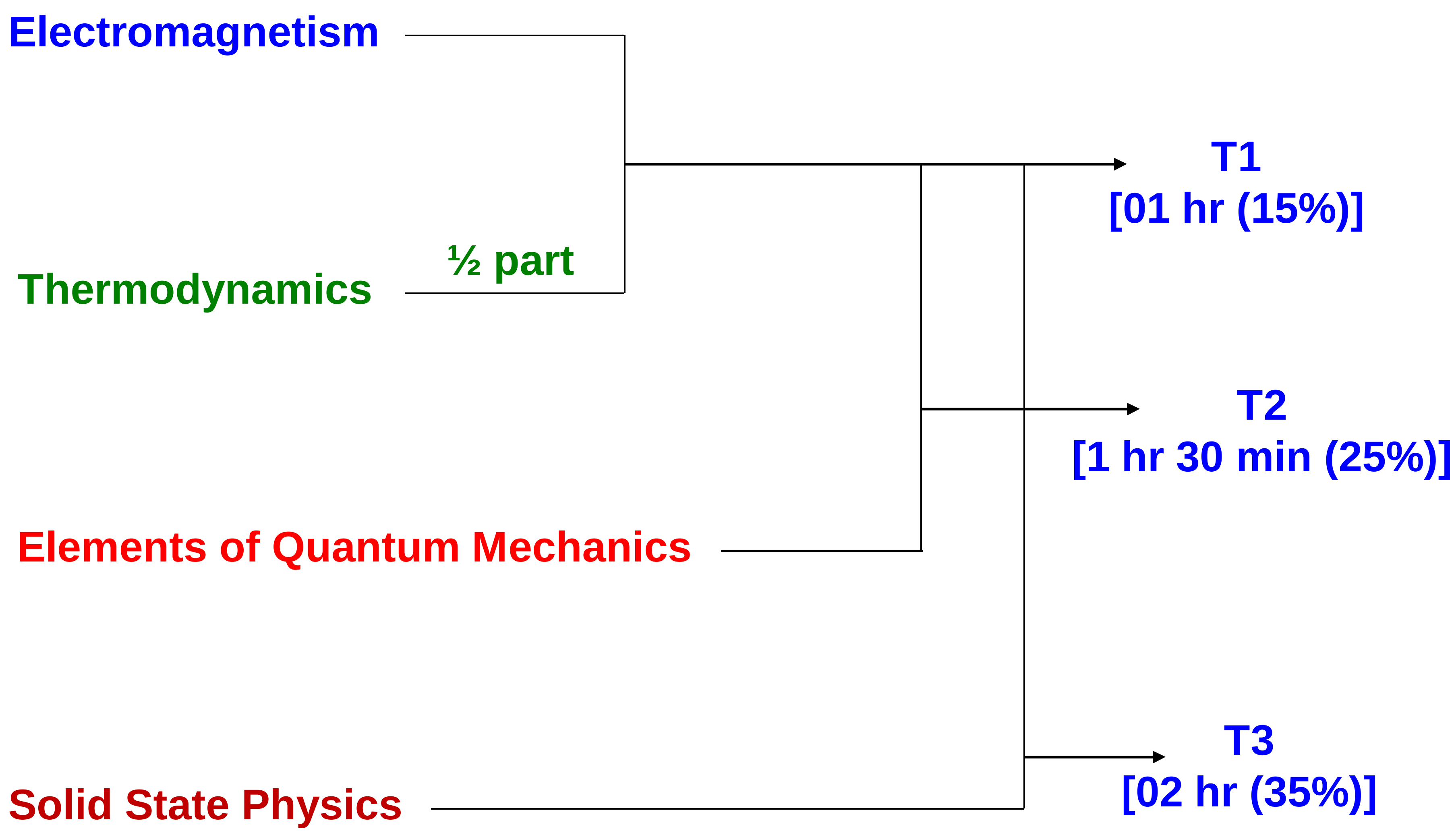
Quantum Mechanics

- Perspectives of Modern Physics, or Concepts of Modern Physics, By: Arthur Beiser
- Schaum's Outline of Theory and Problems of Quantum Mechanics
- Quantum Mechanics By: L.I. Schiff

Solid State Physics

- Perspectives of Modern Physics, or Concepts of Modern Physics, By: Arthur Beiser
- Introduction to Solid State Physics, By: Charles Kittel

Test details



Home assignments, - 10%; Tutorials & Regularity -10%; Attendance - 05%

[80-82:01 marks; 83-85: 02 marks; 86-88: 03 marks; 89: 04 marks; 90 and above: 05 marks]

Scalars and Vectors

A scalar is a number which expresses quantity. Scalars may or may not have units associated with them.

Examples: mass, volume, energy, money

A vector is a quantity which has both magnitude and direction. The magnitude of a vector is a scalar.

Examples: Displacement, velocity, acceleration, electric field

Vector Notation

Vectors are denoted as a symbol with an arrow over the Top and Bold font

$$\vec{A}$$

I $\vec{A}$ **I = Magnitude of vector** $\vec{A}$

$$\vec{A} = A_x\,\hat{a}_x + A_y\,\hat{a}_y + A_z\,\hat{a}_z$$

Where

A_x – Magnitude of Ā in x direction

A_y – Magnitude of Ā in y direction

A_z – Magnitude of Ā in z direction

Modulus or Magnitude of Ā is given by

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

UNIT VECTORS

The unit vector in the Direction of Vector $\vec{A}$ is represented by

$$\hat{a} = \vec{A} \, / \, |\vec{A}|$$

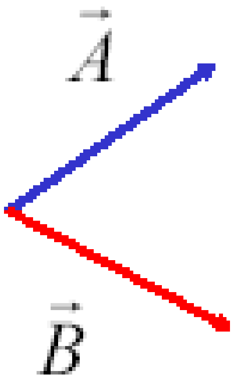
For All unit vectors

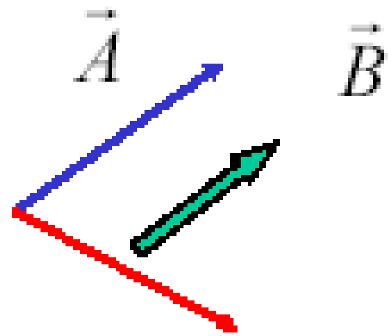
*** Magnitude is Unity**

*** Provide only Direction**

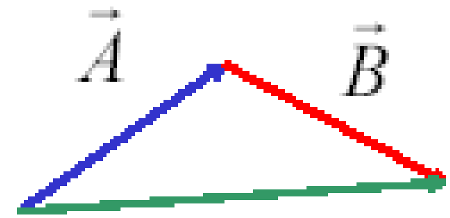
Vector Addition

Adding Vectors Graphically.





Arrange the vectors in a head to tail fashion.

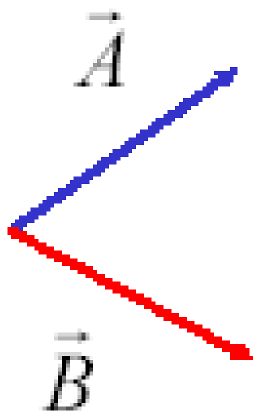


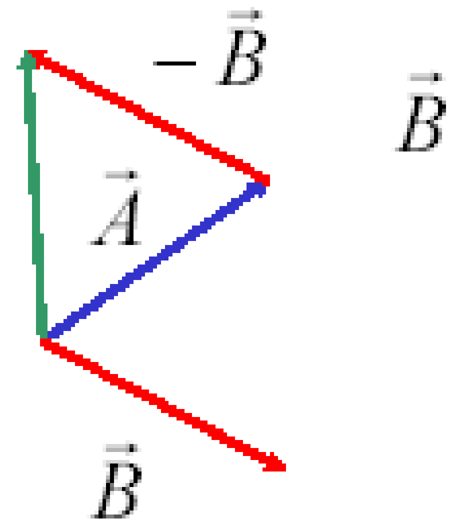
$$\vec{C} = \vec{A} + \vec{B}$$

The resultant is drawn from the tail of the first to the head of the last vector.

Vector Subtraction

Subtracting Vectors Graphically.





$$\vec{C} = \vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$$

Flip one vector.

Then proceed to add the vectors

The resultant is drawn from the tail of the first to the head of the last vector.

Vector Multiplication

- Vector multiplied by a scalar yielding a vector output
- Vector multiplied by a vector yielding a scalar output (Dot product)
- Vector multiplied by a vector yielding a vector output (Cross product)

Vector Multiplication I

The result of vector and scalar is a vector!

Let Ā is multiplied by Scalar k   Then magnitude becomes k times of the Ā

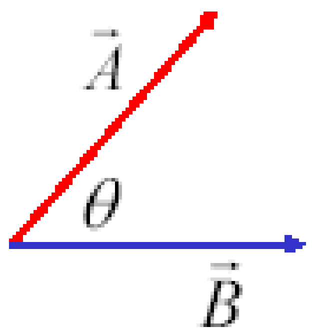
$$\vec{B} = k\vec{A}$$

$k > 0$	+ ve same direction
$k < 0$	-ve opposite direction
$1 < k$	Magnitude increases
$0 < k < 1$	Magnitude decreases

Vector Multiplication II: The Dot Product

The result of a dot product of two vectors is a *scalar*!

$$\boxed{\vec{A}\cdot\vec{B} = AB\cos\theta}$$



θ is an acute angle between the vectors

If $\theta = 0$ then dot product

$$\vec{A}\cdot\vec{B} = AB$$

If $\theta = 90^0$ then dot product

$$\vec{A}\cdot\vec{B} = \boldsymbol{0}$$

Properties

$$\hat{i}\cdot\hat{i} = 1 \qquad \hat{i}\cdot\hat{j} = 0$$

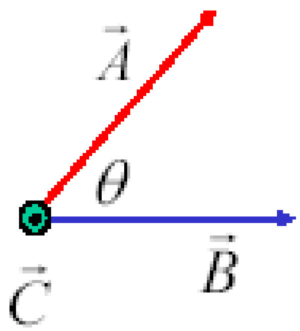
$$\hat{j}\cdot\hat{j} = 1 \qquad \hat{j}\cdot\hat{k} = 0$$

$$\hat{k}\cdot\hat{k} = 1 \qquad \hat{i}\cdot\hat{k} = 0$$

Vector Multiplication III: The Cross Product

The result of a cross product of two vectors is a new *vector*!

$$\boxed{\left|\vec{A}\times\vec{B}\right| = AB\sin\theta}$$



$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

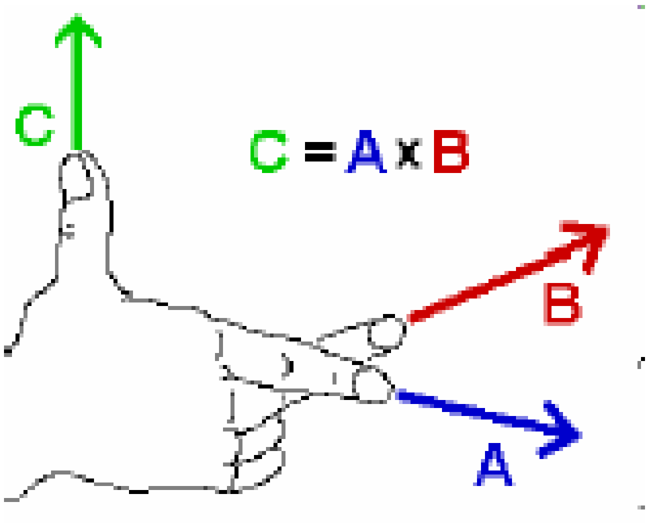
Properties

$$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{i}\times\hat{i} = 0$$

$$\hat{j}\times\hat{k} = \hat{i} \qquad \hat{j}\times\hat{j} = 0$$

$$\hat{k}\times\hat{i} = \hat{j} \qquad \hat{k}\times\hat{k} = 0$$

Vector Multiplication II: Right Hand Rule



Index finger in the direction of the first vector.

Middle finger in the direction of the second vector

Thumb points in the direction of the cross product.

WARNING: Make sure you are using your right hand!!!

Vector Derivatives

First derivatives:

Gradient (∇)

Divergence ($\nabla \bullet$)

Curl ($\nabla \times$)

Second derivatives:

The Laplacian (∇^2) and its relatives

Differential Vector Operator

The **vector derivative operator** ∇ ("del")

$$\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

Differential Vector Operator (cont.)

There are three kinds of vector derivatives, corresponding to the three kind of multiplications possible with vectors:

- **Gradient**, the analogue of multiplication by a scalar.

$$\nabla A$$

- **Divergence**, the analogue of the scalar (dot) product.

$$\nabla \bullet \vec{A}$$

- **Curl**, the analogue to the vector (cross) product.

$$\nabla \times \vec{A}$$

The Gradient

The result of applying the del-operator on a scalar function *A* is called the **gradient of *A***:

$$\nabla A = \left(\frac{\partial A}{\partial x}\right)\widehat{\boldsymbol{x}} + \left(\frac{\partial A}{\partial y}\right)\widehat{\boldsymbol{y}} + \left(\frac{\partial A}{\partial z}\right)\widehat{\boldsymbol{z}}$$

Examples

• If the scalar function φ represents the temperature, Then, $\nabla\phi$ = grad ϕ is temperature gradient or rate of change of temperature with distance

temperature ϕ = Magnitude
$\nabla\phi$ = Magnitude and direction

• Let V represent the potential function then $-\nabla V$ will represent the rate of change of potential with distance.

$$-\nabla V = \bar{E}$$

Ex.1 Given a potential function $V = 2x^2 + 4y$ V in free space find the electric field at the origin.

$$\bar{E} = -\nabla V$$

$$\vec{E} = -\left[\left(\frac{\partial V}{\partial x}\right)\widehat{\boldsymbol{x}} + \left(\frac{\partial V}{\partial y}\right)\widehat{\boldsymbol{y}} + \left(\frac{\partial V}{\partial z}\right)\widehat{\boldsymbol{z}}\right]$$

$$\vec{E} = -\left[4\boldsymbol{x}\ \widehat{\boldsymbol{x}} + 4\widehat{\boldsymbol{y}}\right] \text{V/m}$$

At origin

$$\bar{E} = -4\,\hat{y}\ \text{V/m}$$

The Divergence

The scalar product of the del-operator and a vector function is called the **divergence** of the vector function:

$$\vec{\nabla} \bullet \vec{A} = \left(\widehat{\boldsymbol{x}} \frac{\partial}{\partial x} + \widehat{\boldsymbol{y}} \frac{\partial}{\partial y} + \widehat{\boldsymbol{z}} \frac{\partial}{\partial z} \right) \bullet \left(A_x \widehat{\boldsymbol{x}} + A_y \widehat{\boldsymbol{y}} + A_z \widehat{\boldsymbol{z}} \right) = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

The divergence of a vector function is a scalar!

What is the divergence? Roughly speaking, the divergence is a measure of how fast the field lines stretch and/or spread out.

If two objects following the direction specified by the vector function increase their separation, the divergence of the vector function is positive. If their separation decreases, the divergence is negative.

Ex.2. Electric field density

$$\vec{D} = \left[10xyz\ \hat{\boldsymbol{x}} + 5x^2 \boldsymbol{y}\ \hat{\boldsymbol{y}}\right]$$

Calculate charge density at (1,1,1)

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\rho_v = \left[\left(\frac{\partial}{\partial x}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial}{\partial y}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial}{\partial z}\right)\hat{\boldsymbol{z}}\right] \bullet \vec{D}$$

$$= \left[\left(\frac{\partial}{\partial x} 10xyz\right) + \left(\frac{\partial}{\partial y} 5x^2 y\right)\right] \bullet \vec{D} \qquad = 10\ yz + 5\ x^2$$

At (1,1,1) $= 15$ c/m^3 Diverge

At (0,0,0) $= 0$ c/m^3 neither diverge nor converge

At (1,-1,1) $= -5$c/m^3 Converge

The Curl

The curl of a vector function $\boldsymbol{A}$ is

$$\vec{\nabla}\times\vec{A}=\begin{vmatrix}\widehat{\boldsymbol{x}} & \widehat{\boldsymbol{y}} & \widehat{\boldsymbol{z}}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ A_x & A_y & A_z\end{vmatrix}=\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)\widehat{\boldsymbol{x}}+\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)\widehat{\boldsymbol{y}}+\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)\widehat{\boldsymbol{z}}$$

The curl of a vector function ***A*** is a vector.

Roughly speaking, the curl is a measure of how fast the field-lines of a vector field twist or bend in a direction set by the right-hand rule It is also denote the rate of rotation of field vector at particular point.

The rotation will always be anticlockwise when the ∇ x $\vec{A}$ is + ve

The rotation will always be clockwise when the ∇ x $\vec{A}$ is – ve

There is no rotation is ∇ x $\vec{A}$ is = 0

Ex. 3. Given $A = \left[-y\,\widehat{\boldsymbol{x}} + x\,\widehat{\boldsymbol{y}} \right]$. Find the curl A ?

$$\vec{\nabla} \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \widehat{\boldsymbol{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \widehat{\boldsymbol{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \widehat{\boldsymbol{z}}$$

$$\vec{\nabla} \times \vec{A} = 2\widehat{\boldsymbol{z}}$$

This function has a + ve curl so rotation will be anticlockwise.

Physical Interpretation

Gradient : Maximum space rate change

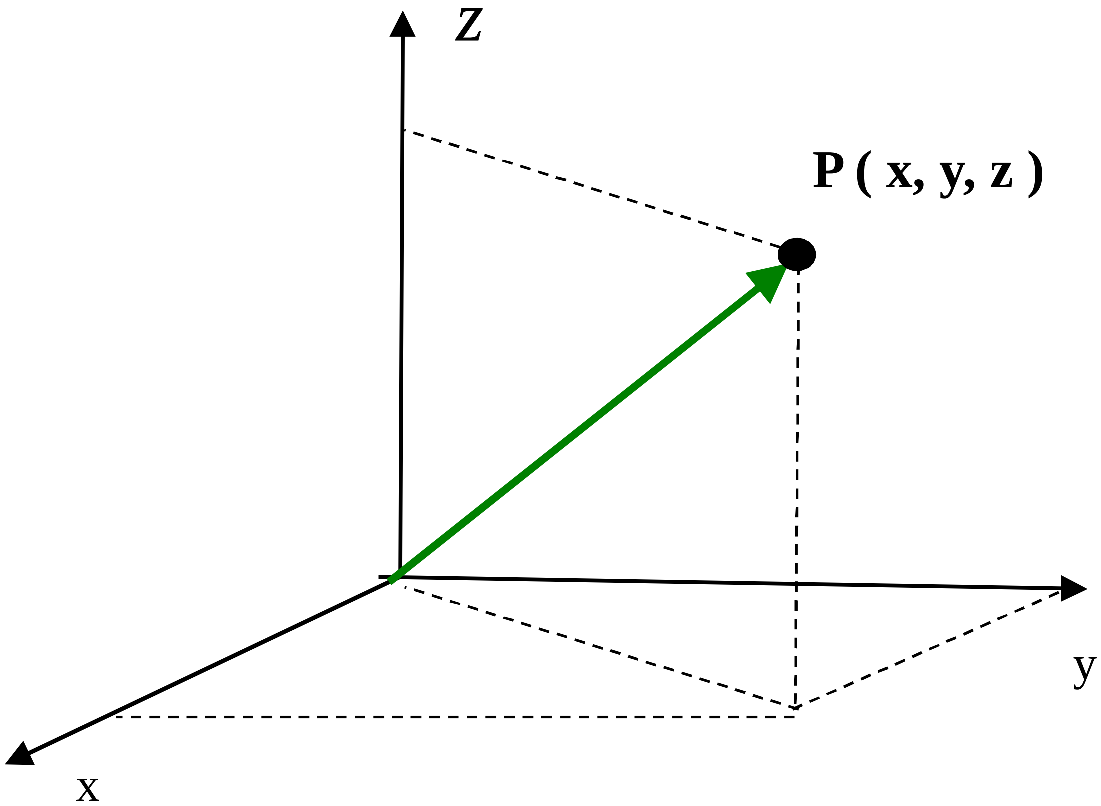
Divergence :
Rate of separation diverging or converging field

Curl : Rotation of field

Different Coordinate Systems

- Cartesian (Rectangular) Coordinate System
- Cylindrical Coordinate System
- Spherical Coordinate System

Cartesian Coordinate System



Coordinates x, y, z

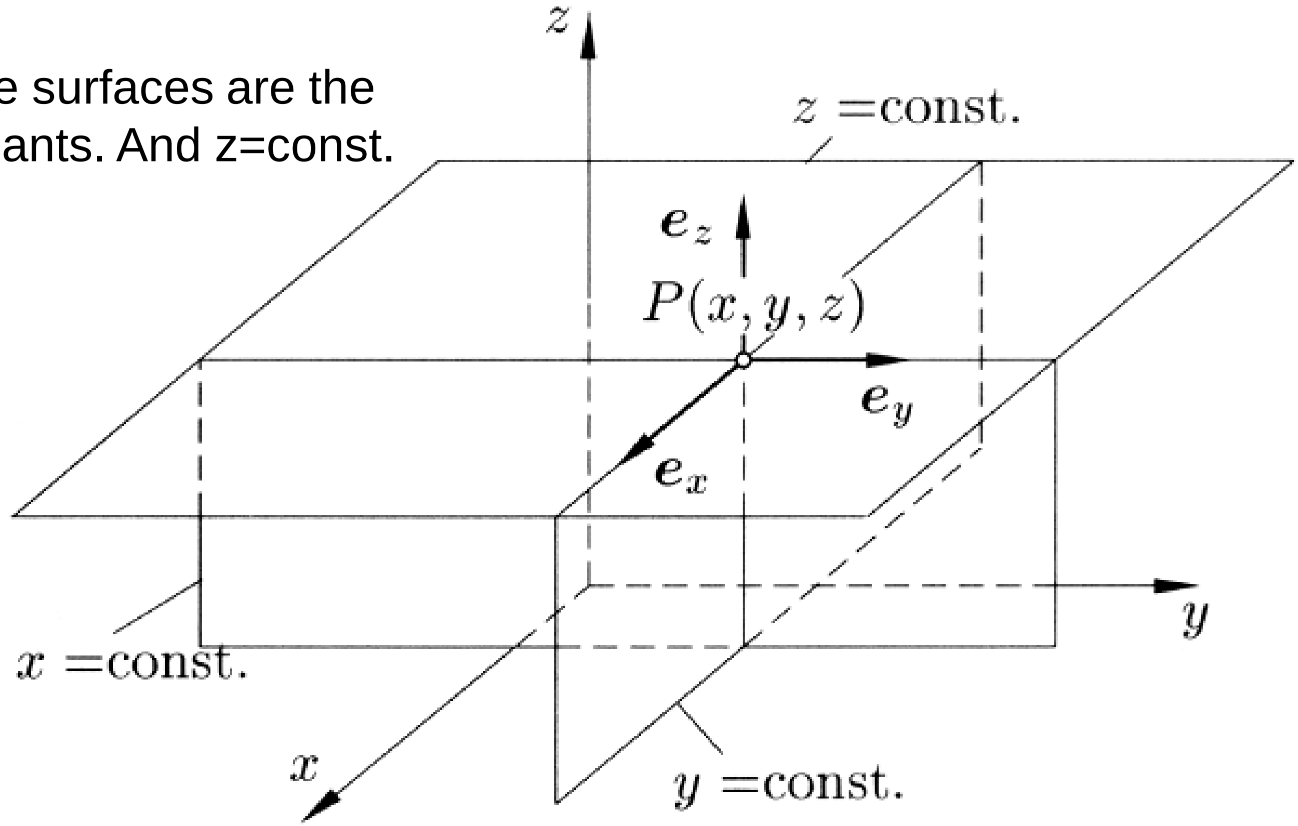
Cartesian Coordinate System (cont.)

• A point is also defined by the intersection of three orthogonal surfaces.

• In cartesian coordinates the surfaces are the infinite planes x= const., y=cants. And z=const.

Limits

$-\infty < x < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$



Unit vectors have fixed directions, independent of the location of point P

Differential Volume, Surface and Line elements

Cartesian coordinates system

Differential elements - dx, dy, dz

Volume $dV = dx\, dy\, dz$

Area

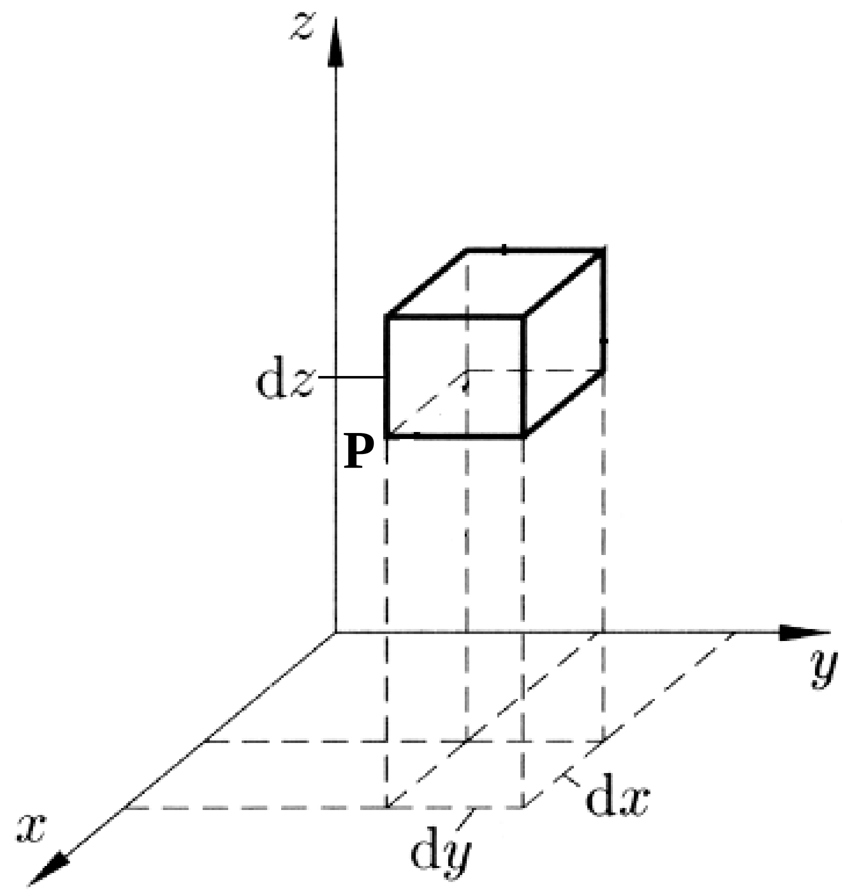
x const. $dA_1 = dy\, dz$

y const. $dA_2 = dx\, dz$

z const. $dA_3 = dx\, dy$

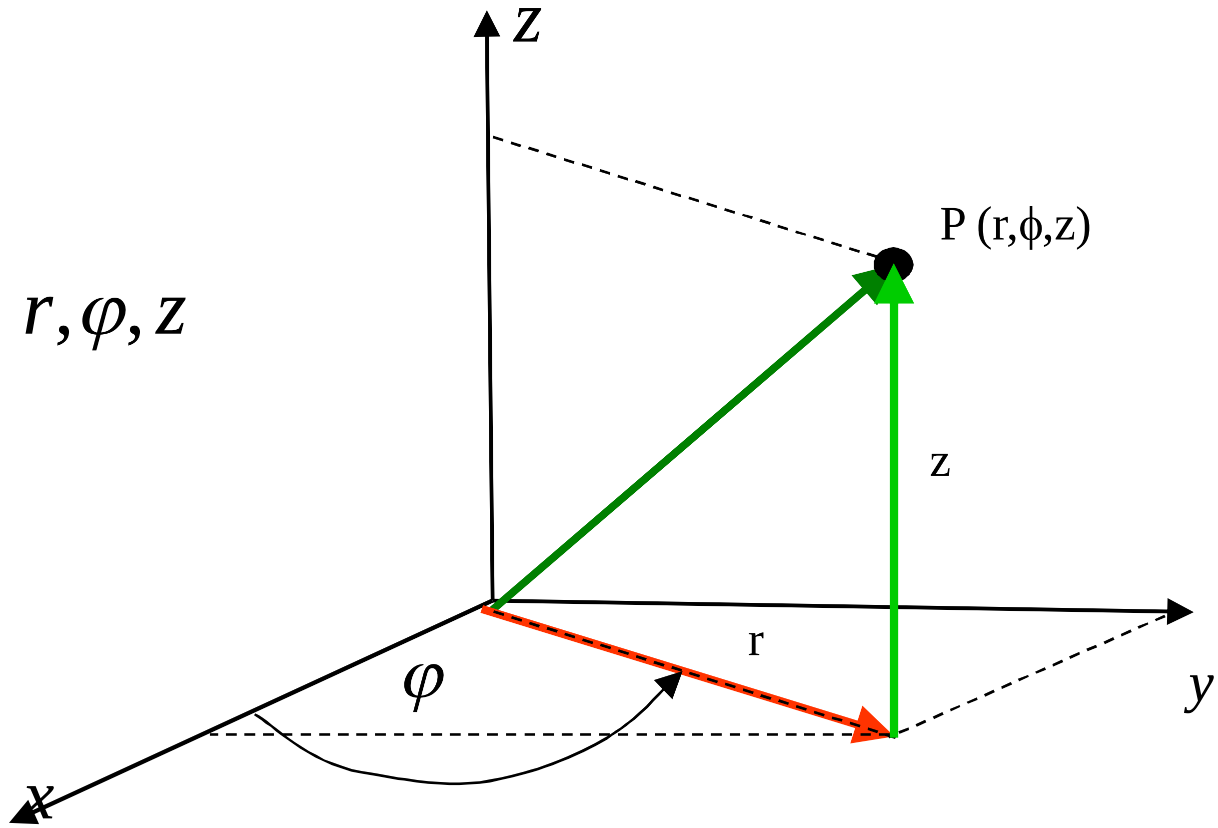
Differential line elements

$$dl^2 = dx^2 + dy^2 + dz^2$$



Cylindrical Coordinate System

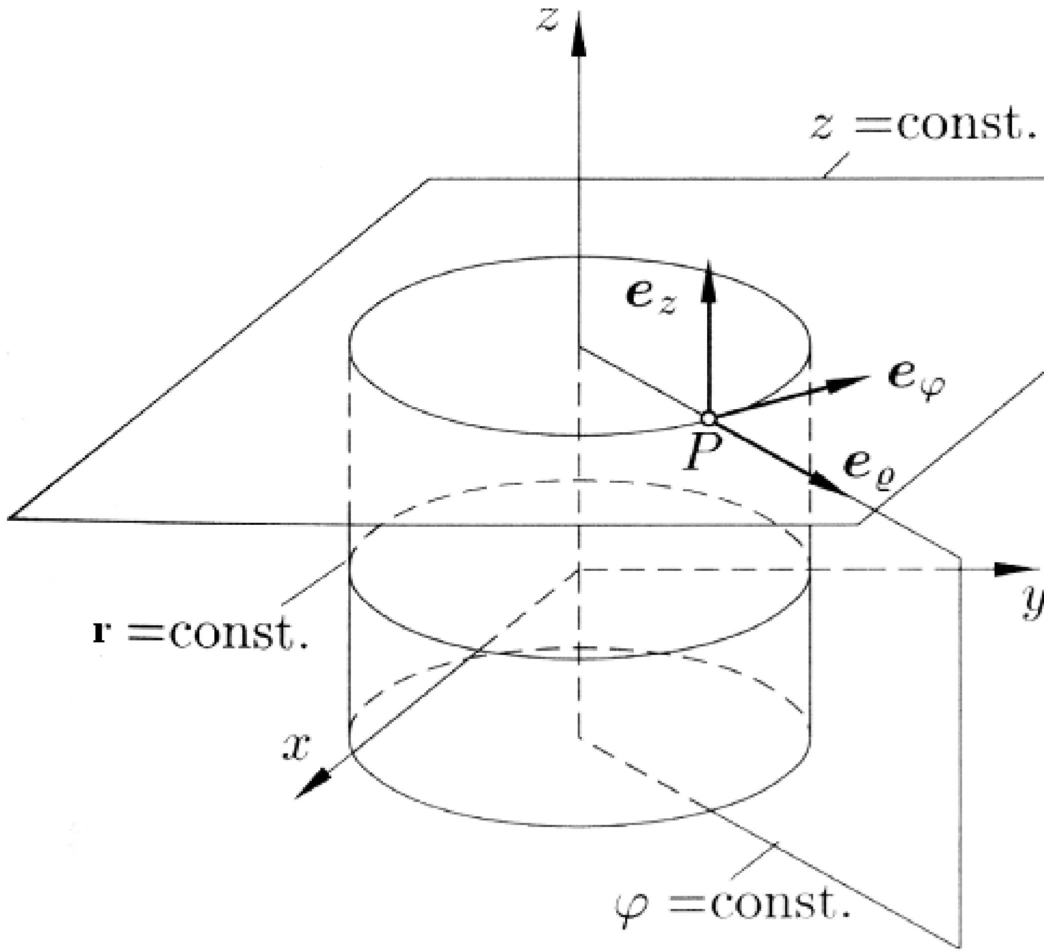
Coordinates r, φ, z



- r is the distance from the z axis in a plane normal to the z axis
- ϕ is the angle between the x axis and the projection of point P on the xy plane
- z is the height of the cylinder

Cylindrical Coordinate System (cont.)

- z = const. is the infinite plane
- ϕ = const. is a half plane with its edge along the z axis
- r = const. is a right circular cylinder



Limits

$$0 \le r < \infty$$
$$0 \le \varphi < 2\pi$$
$$-\infty < z < \infty$$

Each unit vector is normal to its coordinates surface and is in the direction in which the coordinate increases

Differential Volume, Surface and Line elements

Cylindrical Coordinate System

Differential elements - dr , $r d\phi$, dz

Volume $dV = r\,dr\,d\phi\,dz$

Areas

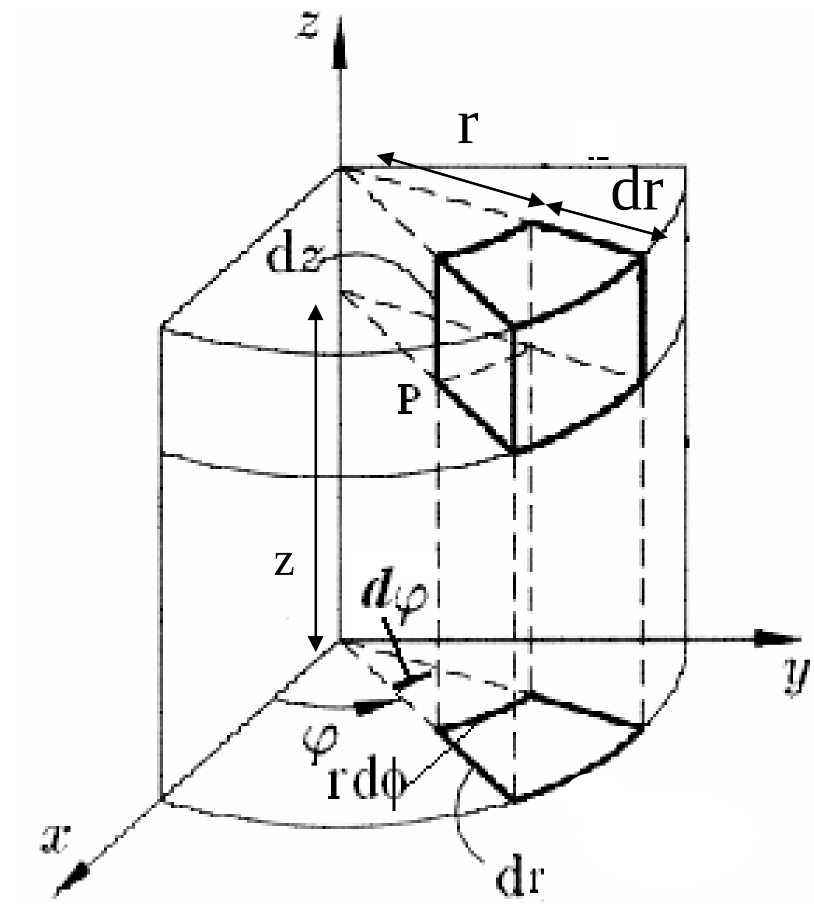
dr const. $dA_1 = r d\phi\, dz$

$d\phi$ const. $dA_2 = dr\, dz$

dz const. $dA_3 = r d\phi\, dr$

Differential line elements

$$dl^2 = dr^2 + rd\phi^2 + dz^2$$



Example : 4

Use cylindrical coordinates to find the area of the curved surface of a right circular cylinder of radius a and height h

The differential surface element is $dA = r d\phi\, dz$

Then
$$A = \int_0^h \int_0^{2\pi} r d\phi dz$$

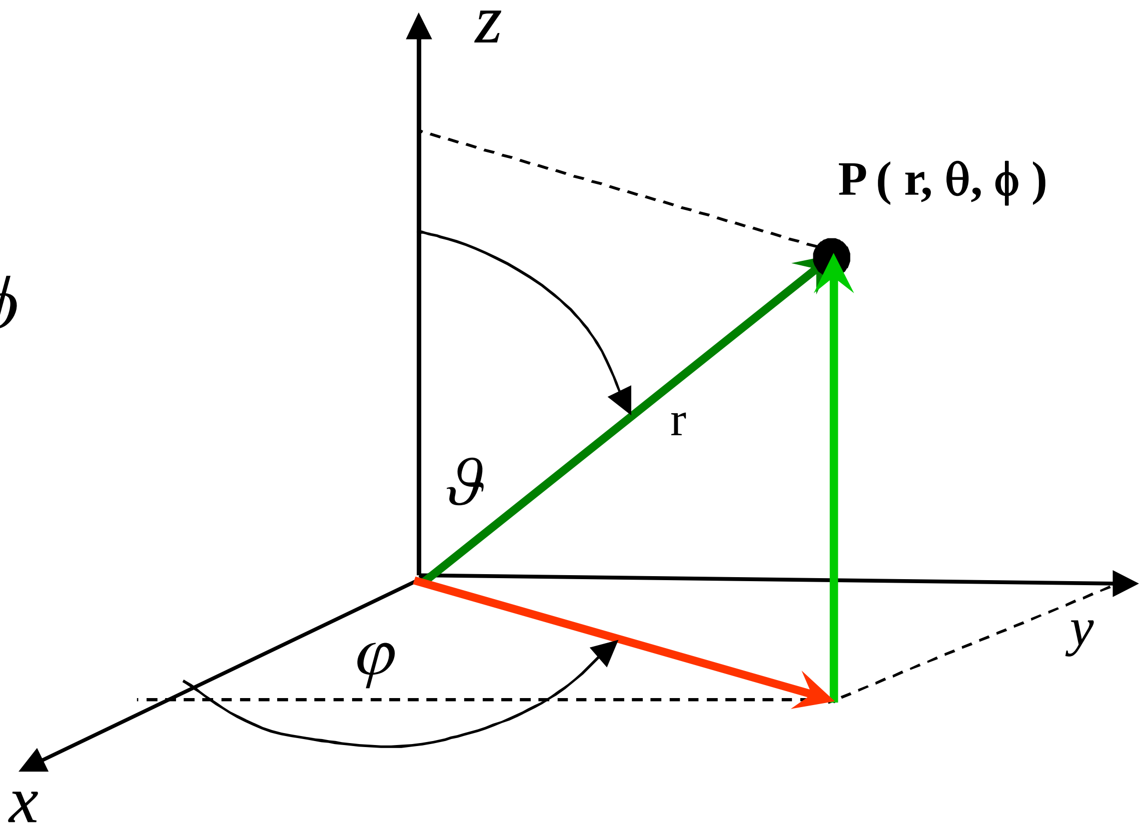
$$A = a\int_0^h \int_0^{2\pi} d\phi dz = 2\pi ah$$

Its volume (for a radius $r = a$) is
$$V = \int_0^a \int_0^h \int_0^{2\pi} r dr d\phi dz$$

$$V = \pi a^2 h$$

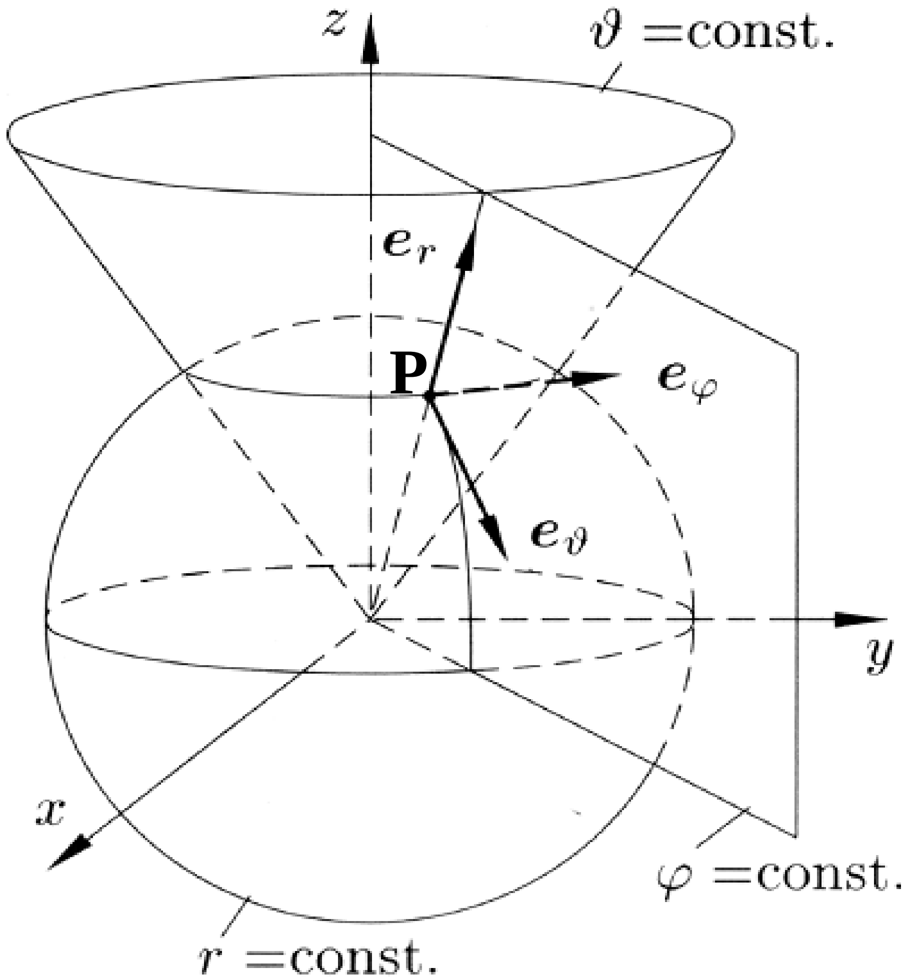
Spherical Coordinate System

Coordinates r, θ, ϕ



- **r is the distance from origin to the point**
- **ϕ is the angle between the x axis and the projection of point P on the xy plane**
- **θ is the acute angle formed from z axis to the OP**

Spherical Coordinate System (cont)



Limits

$$0 \leq r < \infty$$
$$0 \leq \vartheta \leq \pi$$
$$0 \leq \varphi < 2\pi$$

- **ϕ = const. is a half plane with its edge along the z axis**
- **r = conts. is a sphere with center at origin**
- **θ = const. is a right circular cone whose axis is the z axis and whose vertex is at the origin**

Differential Volume, Surface and Line elements

Spherical Coordinate System

Differential elements - dr, rdθ, rsinθ dϕ

Volume $dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$

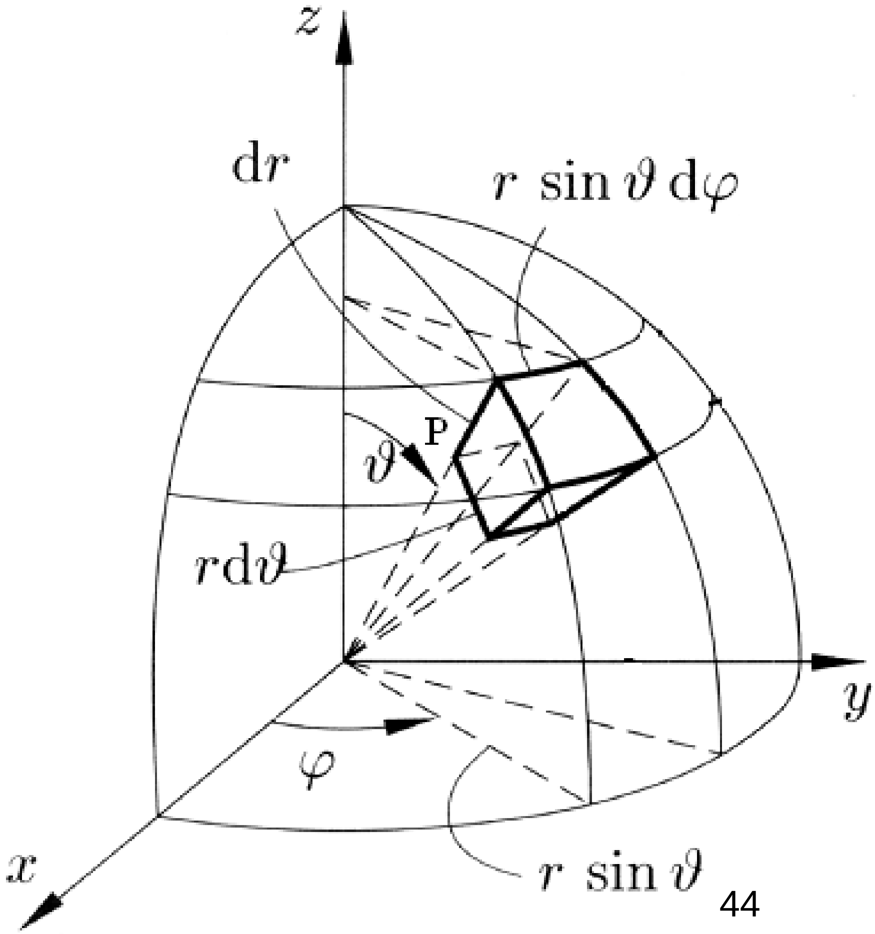
Areas: ϕ const. $dA_1 = r \, d\theta \, dr$

r const. $dA_2 = r^2 \sin\theta \, d\theta \, d\phi$

θ const. $dA_3 = r \sin\theta \, dr \, d\phi$

Differential line elements

$$dl^2 = dr^2 + (r\, d\theta)^2 + (r \sin\theta \, d\phi)^2$$



Example 5

The component of differential area normal to a spherical surface is $\mathbf{a}_r \cdot d\mathbf{s} = r^2 \sin\theta \, d\theta \, d\phi$. Thus, the surface area of a sphere is

The differential surface element is $ds = r^2 \sin\theta d\theta \, d\phi$

$$s = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^2 \sin\theta \, d\theta \, d\phi = 2\pi r^2 \int_0^{\pi} \sin\theta \, d\theta = 4\pi r^2 \quad (\mathrm{m}^2).$$

Its volume (for a radius $r = a$) is

$$v = \int_{r=0}^{a} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^2 \sin\theta \, dr \, d\theta \, d\phi = 4\pi \int_0^{a} r^2 \, dr,$$

$$v = (4/3)\pi a^3 \quad (\mathrm{m}^3).$$

Component forms of vector in the three systems

Cartesian coordinate system

$\bar{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$

Cylindrical coordinate system

$\bar{A} = A_r\hat{a}_r + A_\phi\hat{a}_\phi + A_z\hat{a}_z$

Spherical coordinate system

$\bar{A} = A_r\hat{a}_r + A_\theta\hat{a}_\theta + A_\phi\hat{a}_\phi$

VECTOR OPERATIONS—RECTANGULAR COORDINATES

$$\nabla\alpha = \mathbf{a}_x \frac{\partial \alpha}{\partial x} + \mathbf{a}_y \frac{\partial \alpha}{\partial y} + \mathbf{a}_z \frac{\partial \alpha}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \mathbf{a}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{a}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{a}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

VECTOR OPERATIONS—CYLINDRICAL COORDINATES

$$\boxed{(\nabla A)_r = \frac{\partial A}{\partial r}, \quad (\nabla A)_\phi = \frac{1}{r}\frac{\partial A}{\partial \phi}, \quad (\nabla A)_z = \frac{\partial A}{\partial z}}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \mathbf{a}_\rho\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \mathbf{a}_\phi\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \mathbf{a}_z\frac{1}{\rho}\left(\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi}\right)$$

Vector Operation - Spherical Coordinate System

$$\boxed{(\nabla A)_r = \frac{\partial A}{\partial r}, \quad (\nabla A)_\theta = \frac{1}{r\sin\theta}\frac{\partial A}{\partial \theta}, \quad (\nabla A)_\phi = \frac{\partial A}{r\partial \phi}}$$

$$\nabla \bullet A = a_r \frac{1}{r^2}\frac{\partial\left(r^2 A_r\right)}{\partial r} + a_\theta \frac{1}{r\sin\theta}\frac{\partial\left(\sin\theta A_\theta\right)}{\partial\theta} + a_\phi \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla \times A = a_r \frac{1}{r\sin\theta}\left[\frac{\partial\left(A_\phi \sin\theta\right)}{\partial\theta} - \frac{\partial\left(A_\theta\right)}{\partial\phi}\right] + a_\theta \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial\left(rA_\phi\right)}{\partial r}\right] + a_\phi \frac{1}{r}\left[\frac{\partial\left(rA_\theta\right)}{\partial r} - \frac{\partial\left(A_r\right)}{\partial_\theta}\right]$$

Gauss Divergence Theorem

$$\Rightarrow \quad \oint_S \vec{A}.\mathrm{d}\vec{S} = \underbrace{\int_V \vec{\nabla}.\vec{A}\,\mathrm{d}V}$$

The flux of a vector field $\vec{A}$ on any closed surface S is equal to the volume integral of the divergence of that vector field over that volume enclosed by that surface.

Example 6

Show that the divergence theorem holds for the vector field $\mathbf{A} = \mathbf{a}_r/r$ when the surface is that of a sphere of radius a centered at the origin. We have $\nabla \cdot \mathbf{A} = 1/r^2$ and

$$\oint_S \vec{A}.\mathrm{d}\vec{S} = \int_V \vec{\nabla}.\vec{A}\,\mathrm{d}V$$

$$\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{1}{a}\mathbf{a}_r \cdot \mathbf{a}_r a^2 \sin\theta \, d\theta d\phi = \int_{r=0}^{a}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \sin\theta \, dr \, d\theta d\phi$$

$$4\pi a = 4\pi a.$$

Stokes' Theorem

$$\oint_c \mathbf{A} \cdot d\mathbf{l} = \iint_s (\nabla \times \mathbf{A}) \cdot d\mathbf{s}$$

The surface integral of the curl of a vector field $\bar{A}$ taken over any surface is equal to the line integral of $\bar{A}$ around the closed periphery of the surface

Example 7

Consider the portion of a sphere The surface is specified by $r = 4$, $0 \leq \theta \leq 0.1\pi$, $0 \leq \phi \leq 0.3\pi$, and the closed path forming its perimeter is composed of three circular arcs. We are given the field $\mathbf{H} = 6r \sin\phi \mathbf{a}_r + 18r \sin\theta \cos\phi \mathbf{a}_\phi$ and are asked to evaluate each side of Stokes' theorem.

$$\nabla \times \mathbf{H} = \frac{1}{r \sin\theta}(36r \sin\theta \cos\theta \cos\phi)\mathbf{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta} 6r \cos\phi - 36r \sin\theta \cos\phi\right)\mathbf{a}_\theta$$

The differential path element $d\mathbf{L}$

$$d\mathbf{L} = dr\, \mathbf{a}_r + r\, d\theta\, \mathbf{a}_\theta + r \sin\theta\, d\phi\, \mathbf{a}_\phi$$

$$\oint \mathbf{H} \cdot d\mathbf{L} = \int \left(6r \sin\phi \mathbf{a}_r + 18r \sin\theta \cos\phi \mathbf{a}_\phi\right) \cdot \left[dr\, \mathbf{a}_r + r\, d\theta\, \mathbf{a}_\theta + r \sin\theta\, d\phi\, \mathbf{a}_\phi\right]$$

$$= \int 6r \sin\phi dr + 18r^2 \sin^2\theta \cos\phi d\phi$$

For r = constant dr = 0

$$= \int_0^{0.3\pi} 18r^2 \sin^2\theta \cos\phi d\phi$$

$$\oint \mathbf{H} \cdot d\mathbf{L} = \int_0^{0.3\pi} [18(4) \sin 0.1\pi \cos\phi] 4 \sin 0.1\pi d\phi \qquad = 288 \sin^2 0.1\pi \sin 0.3\pi = 22.2 \text{ A}$$

$$\nabla \times A = a_r \frac{1}{r\sin\theta}\left[\frac{\partial(A_\phi \sin\theta)}{\partial\theta} - \frac{\partial(A_\theta)}{\partial\phi}\right] + a_\theta \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(rA_\phi)}{\partial r}\right] + a_\phi \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial(A_r)}{\partial_\theta}\right]$$

$$\nabla \times \mathbf{H} = \frac{1}{r\sin\theta}(36r\sin\theta\cos\theta\cos\phi)\mathbf{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}6r\cos\phi - 36r\sin\theta\cos\phi\right)\mathbf{a}_\theta$$

Since $d\mathbf{S} = r^2 \sin\theta\, d\theta\, d\phi\, \mathbf{a}_r$, the integral is

$$\int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S} = \int_0^{0.3\pi} \int_0^{0.1\pi} (36\cos\theta\cos\phi)16\sin\theta\, d\theta\, d\phi$$

$$= \int_0^{0.3\pi} 576(\tfrac{1}{2}\sin^2\theta)\Big|_0^{0.1\pi} \cos\phi\, d\phi$$

$$= 288\sin^2 0.1\pi \sin 0.3\pi = 22.2 \text{ A}$$

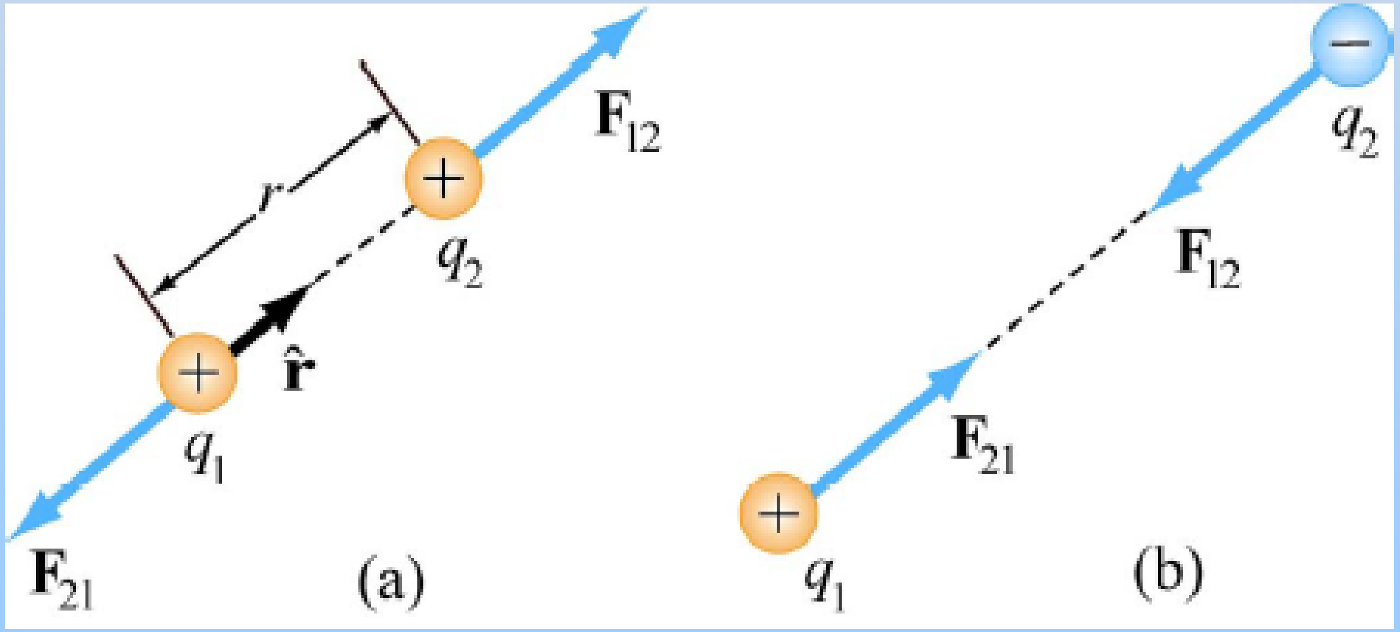
Thus, the results check Stokes' theorem.

Electric Force

The electric force between charges q_1 and q_2 is

(a) repulsive if charges have same signs

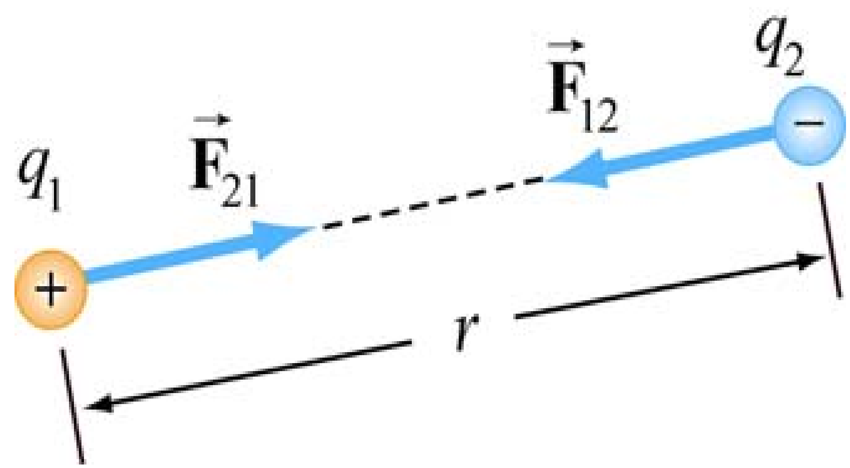
(b) attractive if charges have opposite signs



Like charges repel and opposites attract !!

Coulomb's Law

Force is attractive if charges are opposite sign & repulsive if same.



$$\overrightarrow{F_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\widehat{r}_{12}$$

$$\varepsilon_0 = \frac{1}{36\pi \cdot 10^9} = 8.85 \cdot 10^{-12} \quad \frac{\mathrm{C}^2}{\mathrm{m}^2 \cdot \mathrm{N}}$$

Example 8

Two point charges Q_1 = 50 μc and Q_2 = 10 μc located at (-1,1,-3) m in (3,1,0) m respectively. Find the force on Q_1

$$\overrightarrow{F_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\widehat{r}_{21}$$

r = - 4ax -3az $\qquad \widehat{r} = \dfrac{-4a_x - 3a_z}{5}$

$$\overrightarrow{F_1} = \frac{\left(50\times10^{-6}\right)\left(10^{-5}\right)}{4\pi\left(\dfrac{10^{-9}}{36\pi}\right)(5)^2}\left(\frac{-4a_x - 3a_z}{5}\right) = (.18)\left(-0.8a_x - 0.6a_z\right)N$$

$$\overrightarrow{F_1} = \left(0.144a_x - 0.108a_z\right)N$$

ELECTRIC FIELD INTENSITY

Electric field intensity

$$\mathbf{E} = \mathbf{F} / q$$

ELECTRIC FIELD OF A POINT CHARGE IN VACUUM

Coulomb's law: $$\left|\mathbf{F}\right| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q\,q}{r^2}$$

Intensity of electric field created by the charge Q

$$\boxed{\left|\mathbf{E}\right| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}}$$

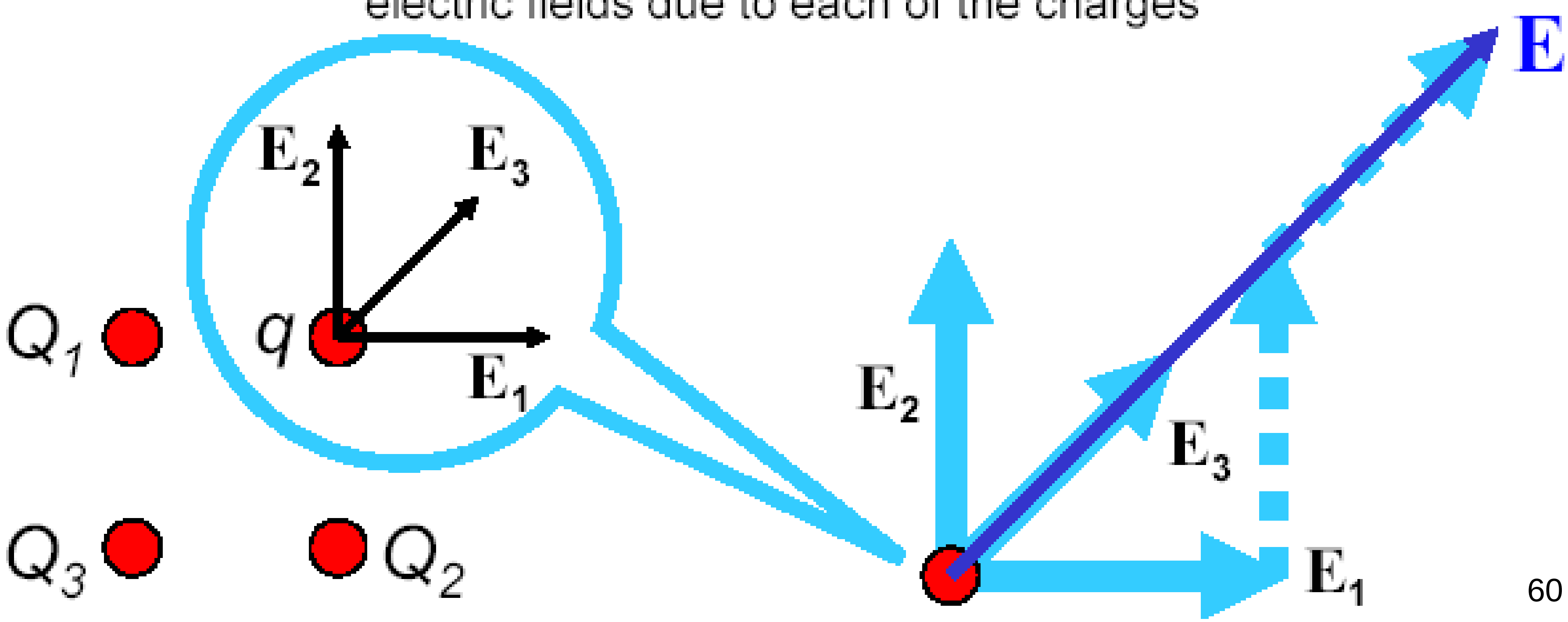
ELECTRIC FIELD FROM MULTIPLE CHARGES: SUPERPOSITION PRINCIPLE

Superposition of forces: $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots$

Therefore, for the electric field intensity

$$\boxed{\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \ldots}$$

Electric field due to multiple charges Q_1, Q_2, Q_3, etc is a **vector** sum of the electric fields due to each of the charges



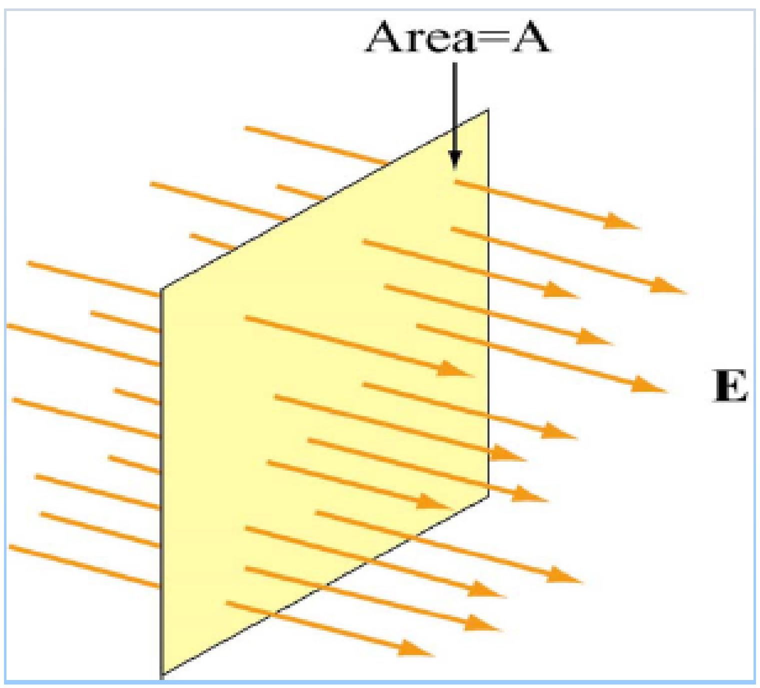
Electric Flux Φ_E

* Flux is a measure of the number of field lines passing through an area

* Electric flux is the number of Electric field lines penetrating a surface or an area.

Total Electric flux passing through the total surface

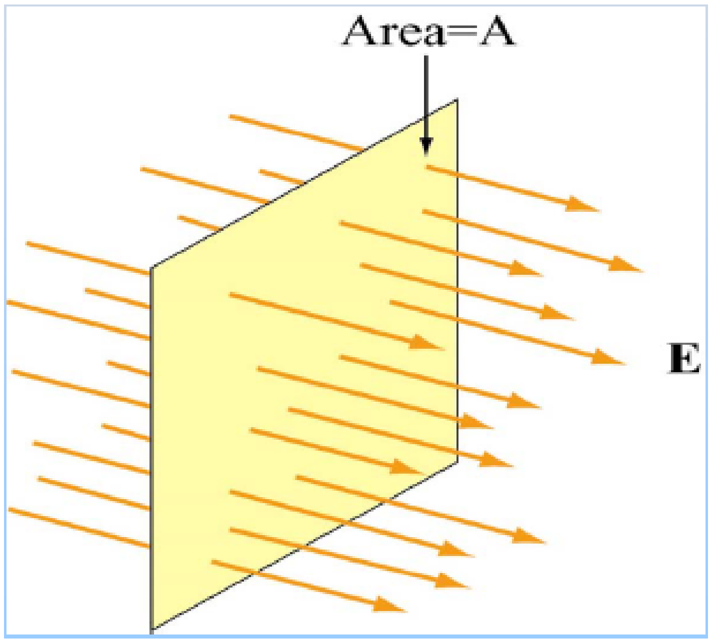
$$\Phi_E = \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$



Electric field lines passing through a surface of area *A*, whose normal makes an angle θ with the field.

$$\text{Electric Flux} = \Phi = (E\cos\theta)A = \vec{E} \cdot \vec{A}$$

Case I: E is constant vector field perpendicular to planar surface S of area A



$$\Phi_E = \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

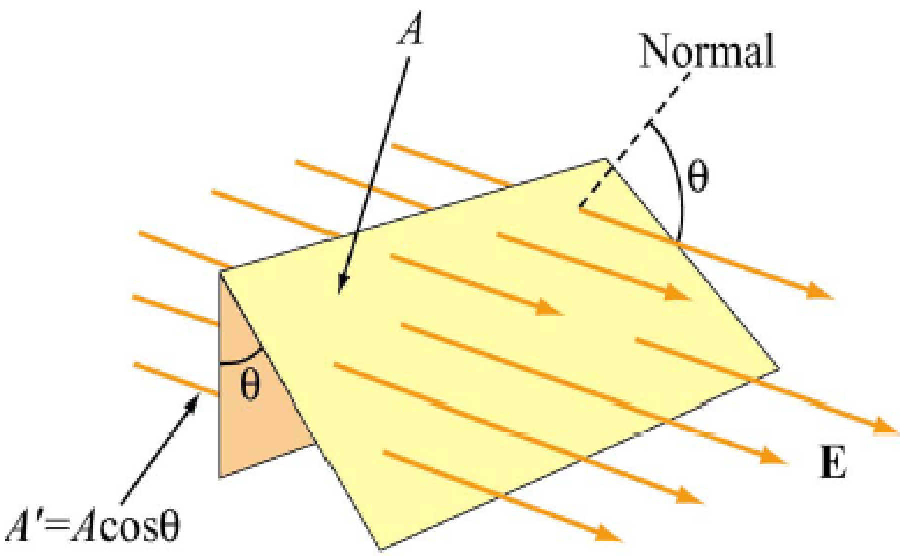
$$\text{Electric Flux} = \Phi = (E\cos\theta)A = \vec{E} \cdot \vec{A}$$

$$\bar{\mathbf{E}} \parallel \bar{\mathbf{A}} \Rightarrow \theta = 0$$

$$\Phi_E = +EA$$

Case II: E is constant vector field directed at angle θ to planar surface S of area A

Electric field lines passing through a surface of area *A* whose normal makes an angle θ with the field.



$\bar{\mathbf{E}}$ И $\bar{\mathbf{A}} \Rightarrow \theta \neq 0$

$$\mathbf{A}' = \mathbf{A}\cos\theta$$

Where A′ is the perpendicular area to the field E

$$\Phi_E = \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\Phi_E = EA\cos\theta$$

The value of electric flux is +ve if lines of forces are diverging
The value of electric flux is -ve if lines of forces are converging

Example 9

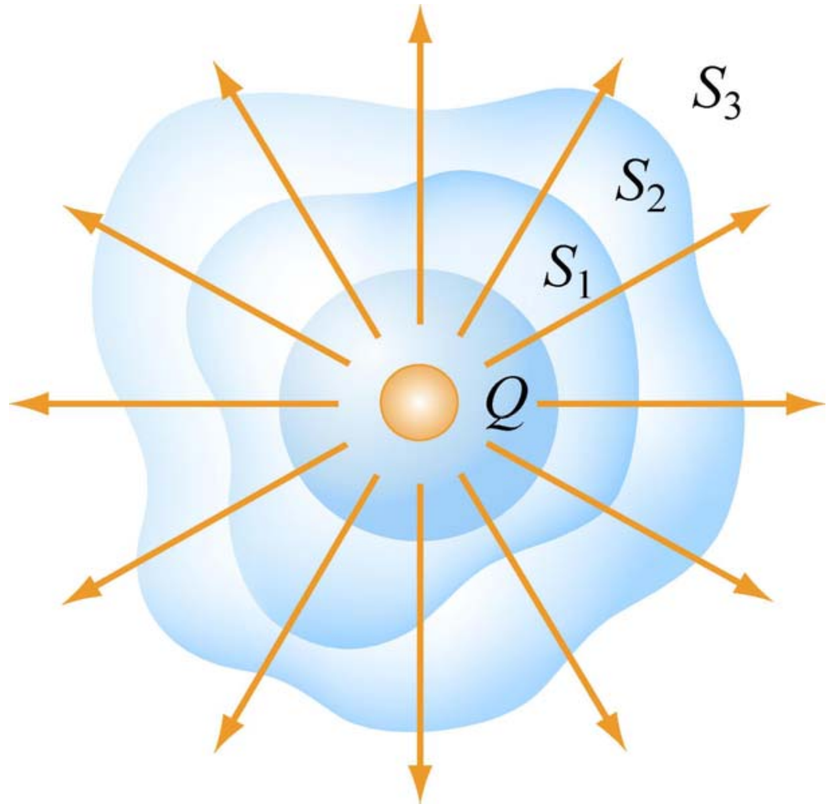
Find the flux of the vector field $\mathbf{A} = \mathbf{a}_r / r^2$ out of the sphere $r = a$, $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$. We have

$$\text{Flux} = \oiint_s \mathbf{A}\Big|_s \cdot d\mathbf{s}$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{1}{r^2}\mathbf{a}_r\Big|_{r=a} \cdot \mathbf{a}_r a^2 \sin\theta \, d\theta \, d\phi,$$

$$\text{Flux} = \int_0^{\pi} \int_0^{2\pi} \sin\theta \, d\theta \, d\phi = 4\pi.$$

Gauss's Law – The Idea



The total "flux" of field lines penetrating any of these surfaces is the same and depends only on the amount of charge inside

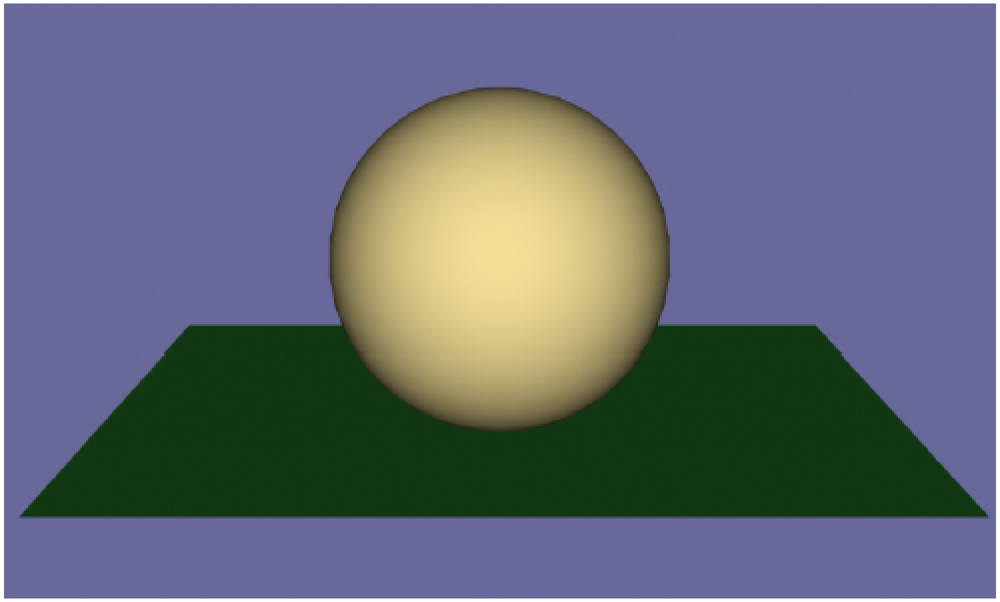
Gauss's Law – The Equation

$$\Phi_E = \underset{\text{closed surface}S}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$$

Electric flux ϕ_E (the surface integral of E over closed surface *S*) is proportional to charge inside the volume enclosed by *S*

Note: Integral must be over closed surface

Open and Closed Surfaces

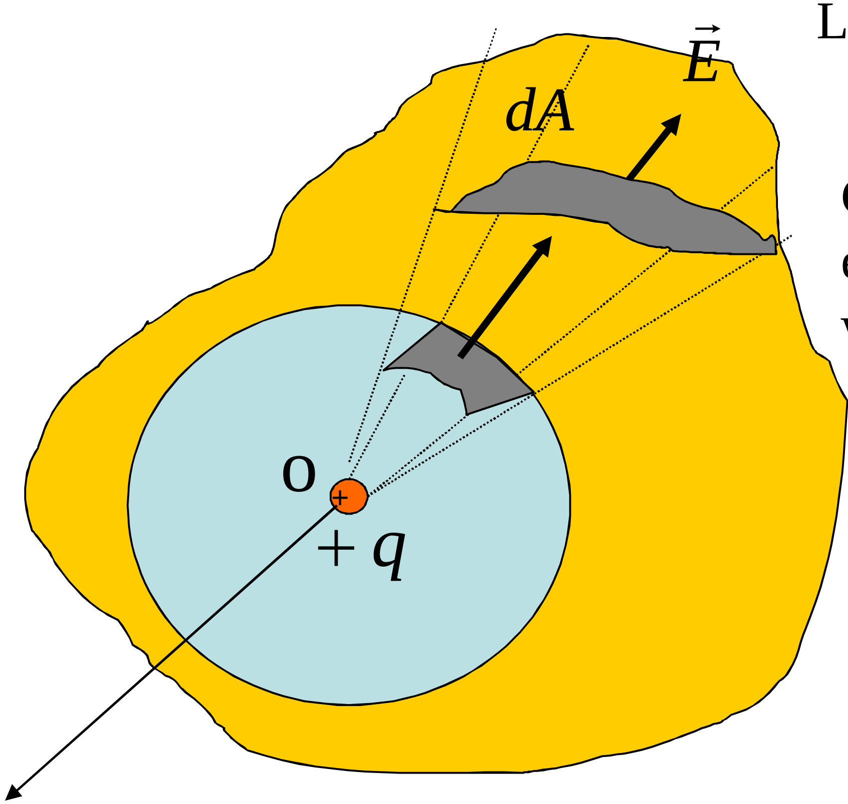


A rectangle is an open surface — it does NOT contain a volume

A sphere is a closed surface — it DOES contain a volume

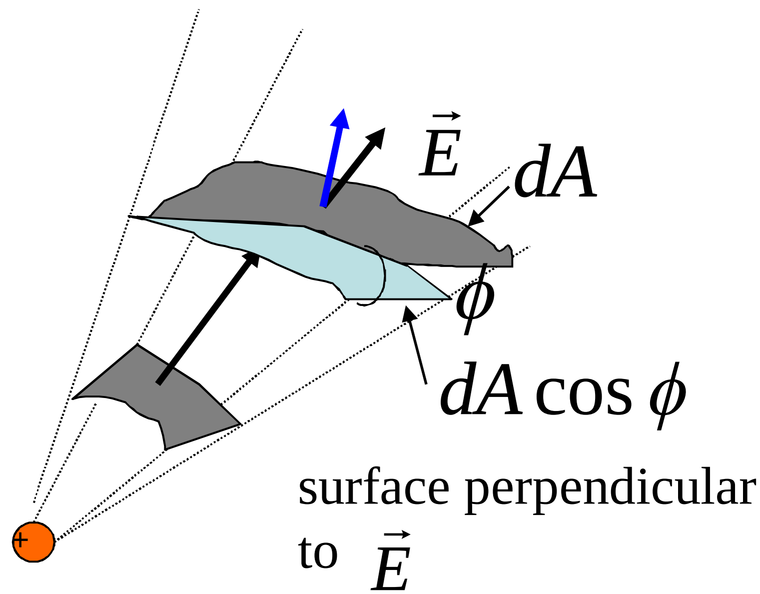
Proof of Gauss's Theorem

- Case 1: Single positive charge inside closed surface



Let a point charge q is placed in the closed surface

Consider a small area dA of the surface. Then the electric flux passing through surface area dA whose normal makes an angle ϕ with the field



Then the electric flux passing through surface area dA whose normal makes an angle ϕ with the field

$$d\Phi_E = E\cos\phi dA \qquad \text{------- (1)}$$

$$d\Phi_E = \frac{q}{4\pi\varepsilon_0 r^2}\cos\phi dA \qquad \text{------- (2)}$$

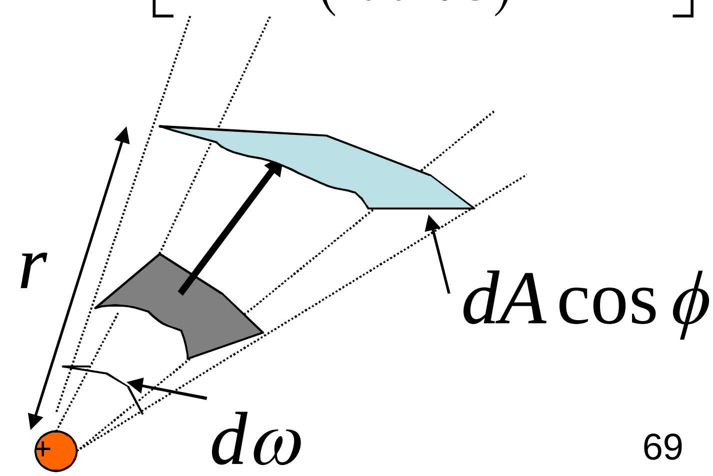
By definition solid angle subtended by a area dA at point O

$$d\omega = \frac{dA\cos\phi}{r^2}$$

$$\left[d\omega = \frac{surface\ \ area}{(radius)^2} \right]$$

Therefore eq 2 reduces to

$$d\Phi_E = \frac{q}{4\pi\varepsilon_0} d\omega$$



Hence the electric flux through whole of the closed surface

$$\Phi_E = \frac{q}{4\pi\varepsilon_0} \oint d\omega$$

(Solid angle subtended by the entire closed surface at an internal point O is 4π **)**

$$\Phi_E = \frac{q}{4\pi\varepsilon_0} \cdot 4\pi$$

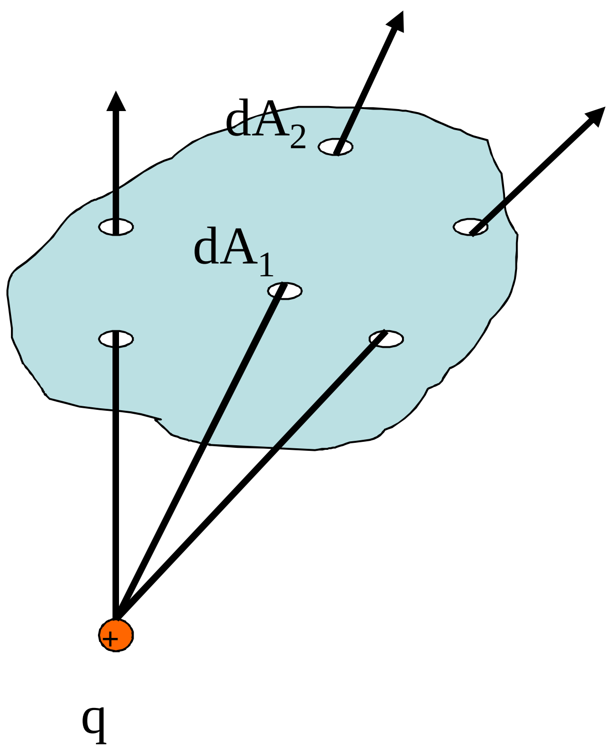
$$\Phi_E = \frac{q}{\varepsilon_0}$$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Hence the total electric flux through any closed surface is equal to $1/\varepsilon_0$ times of total charge enclosed within the surface which is Gauss law.

Gauss's Law

- Case 2: Single positive charge outside closed surface



Electric flux through area dA_1

$$d\Phi_{A_1} = -\frac{q}{4\pi\varepsilon_0} d\omega \qquad \text{---(1)}$$

Electric flux through area dA_2

$$d\Phi_{A_2} = \frac{q}{4\pi\varepsilon_0} d\omega \qquad \text{---(2)}$$

Total electric flux through a closed surface

$$d\Phi = d\Phi_{A_1} + d\Phi_{A_2} \qquad \text{---(3)}$$

Electric field lines that go in come out.
Electric field lines can begin or end inside a region of space only when there is charge in that region.

Putting the value of dA_1 and dA_2 from equation (1) and (2) in eq (3)

$$d\Phi = \frac{q}{4\pi\varepsilon_0} d\omega + \left[-\frac{q}{4\pi\varepsilon_0} d\omega \right]$$

$$\Phi_E = 0$$

As there is no charge within the surface, the total electric flux through the whole surface is zero

Applying Gauss's Law

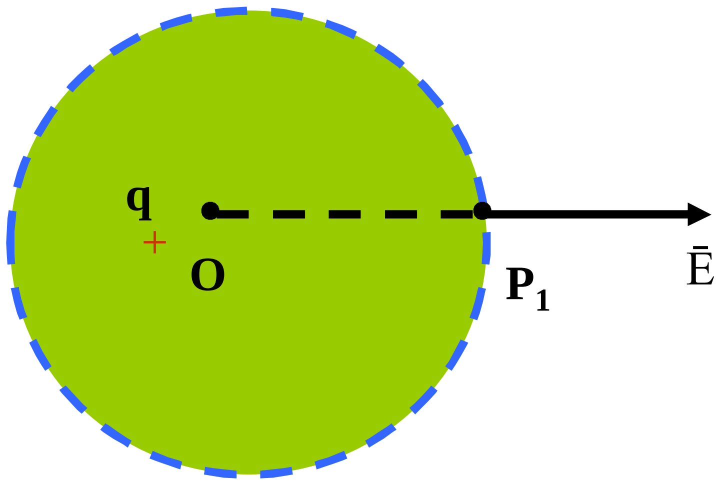
1. Identify regions in which to calculate E field.
2. Choose Gaussian surfaces S: Symmetry
3. Calculate $\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$
4. Calculate q_{in}, charge enclosed by surface S
5. Apply Gauss's Law to calculate E:

$$\boxed{\Phi_E = \oiint_{\substack{\text{closed}\\ \text{surfaceS}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}}$$

Applications of gauss's theorem

(a) Cases of spherical symmetry

(i) Field due to point charge



Electric flux through the spherical surface

$$\phi_E = \oint_A E \cdot dA = EdA = E(4\pi r^2)$$

Charge enclosed by surface $Q_{encl} = q$

By Gauss theorem

$$\oint_A E \cdot dA = \frac{Q_{encl}}{\varepsilon_0}$$

$$E.4\pi r^2 = \frac{1}{\varepsilon_0} \times q$$

$$E = \frac{1}{4\pi r^2 \varepsilon_0} \times q$$

- Electric field E at each point of surface is same & directed outward

(ii) Electric Field due to a charged spherical shell

Case(i) *Electric Field outside the shell*

(1) $r > r_0$:

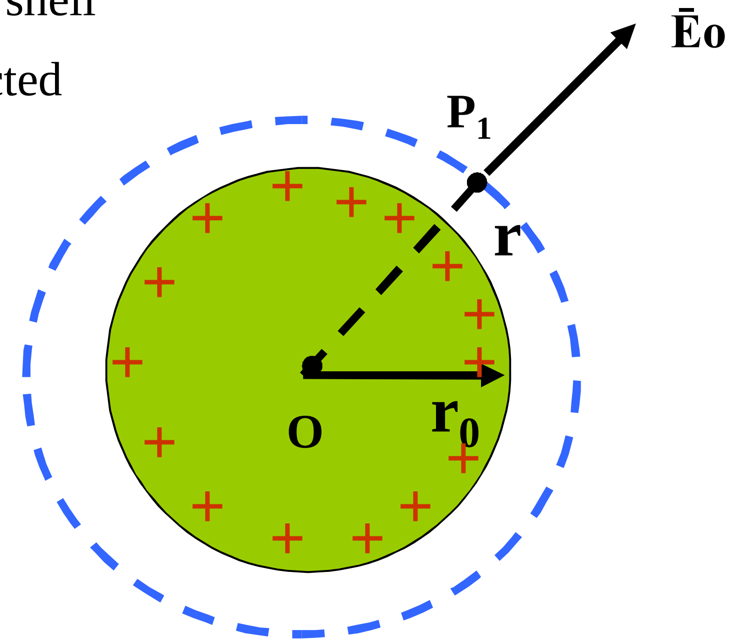
•Spherical shell of radius r_0, carrying a charge Q with centre O

•Imagine a spherical shell of radius r concentric with the shell

•Electic field E_o at each point of surface is same & directed outward

•Let the electric field at the surface be E_o

•Net Electric flux through the whole surface



$$\phi_E = \oint_A E_o \cdot d\underline{A} = E_o(4\pi r^2)$$

By Gauss theo rem

$$\oint_A E_o \cdot d\underline{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Charge enclosed by surface $Q_{encl} = Q$

$$E_o = \frac{Q}{4\pi\varepsilon_0 r^2}$$

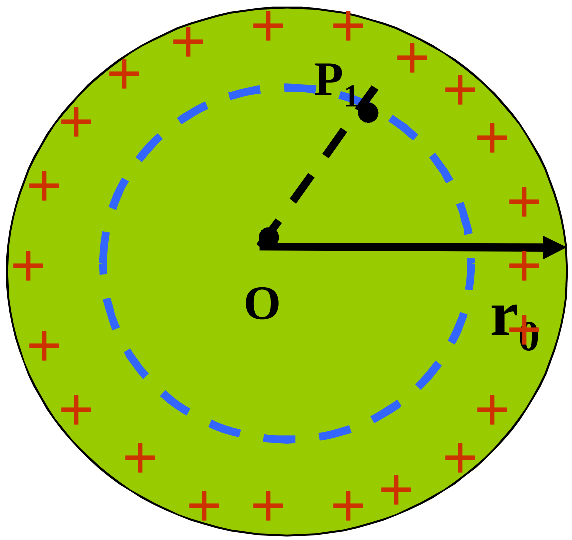
Hence the electric field strength at any pt outside a charged spherical shell is same as through the charge were placed at the centre O.

Case(ii) *Electric Field inside the shell ($r < r_0$)*

If E_i is the electric field inside the shell, then by symmetry E_i is same at each point of spherical surface and is directed outward

$$\oint_A E_i \cdot d\underline{A} = E_i(4\pi r^2)$$

Net charge enclosed by spherical surface $Q_{encl} = 0$



By Gauss theorem

$$\oint_A E_i \cdot d\underline{A} = \frac{Q_{encl}}{\varepsilon_0}$$

$$E_i 4\pi r^2 = \frac{1}{\varepsilon_0} \times 0$$

$$E_i = 0$$

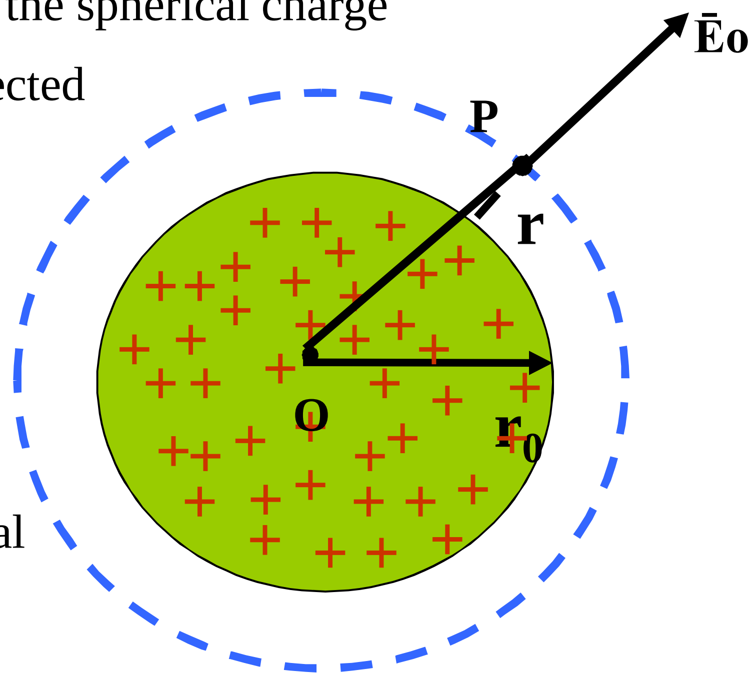
Thus electric field strength at each point within the shell is zero

(ii) Electric Field due to a spherically charge distribution

Case(i) *Electric Field strength at an external point*

(1) $r > r_0$:

- Spherical charge distribution of radius r_0, carrying a charge Q with centre O
- Imagine a spherical surface of radius r concentric with the spherical charge
- Electic field E_o at each point of surface is same & directed outward
- Let the electric field at the surface be E_o
- Net Electric flux through the whole surface



$$\oint_A E_o \cdot d\underline{A} = E_o(4\pi r^2)$$

By Gauss theorem the total charge enclosed by the spherical surface = Q

$$\oint_A E_o \cdot d\underline{A} = \frac{Q_{encl}}{\varepsilon_0}$$

$Q_{encl} = Q$

$$E_o = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$E_i \; \alpha \; 1/r^2$

Hence the electric field strength at any pt outside a spherical charge distribution is the same as through the whole charge were concentrated at the center.

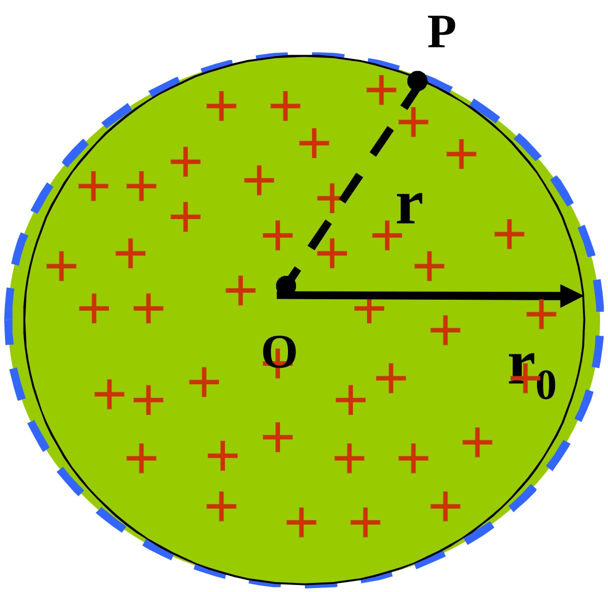
Case(ii) *Electric Field strength on the surface of the spherical charge distribution*

(2) r = r$_0$

•In this case the distance of point P from the center of the charge distribution is equal to its radius

•Electric flux through the whole surface

$$\oint_A E \cdot d\underline{A} = E_o(4\pi r_0^{\ 2})$$



By Gauss theorem the total charge enclosed by the spherical surface = Q

$$\oint_A E \cdot d\underline{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Electric Field strength on the surface of the spherical charge distribution

$$E = \frac{Q}{4\pi\varepsilon_0 r_0^{\ 2}}$$

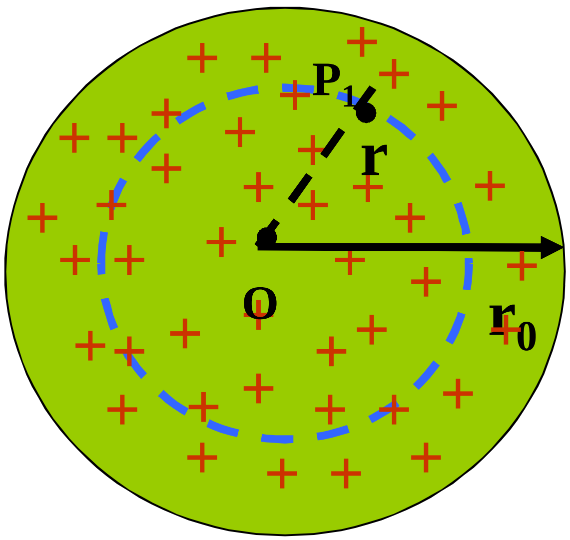
$E_i \ \alpha \ 1/\ r_0^{\,2}$

Case(iii) *Electric Field strength inside the charge distribution*

(3) $r < r_0$

Consider a spherical surface of radius r concentric with spherical charge
Let ρ be the volume charge density (charge per unit volume) of uniform distribution of spherical charge

$$\rho = \frac{charge}{volume} = \frac{Q}{\frac{4}{3}\pi r_0^{\,3}}$$



•Total electric flux through the whole surface

$$\oint_A E_i \cdot d\underline{A} = E_i(4\pi r^2)$$

Net charge enclosed by the Gaussian surface $= \frac{4}{3}\pi r^3 \rho$

By Gauss theorem

$$\oint_A E_i \cdot d\underline{A} = \frac{4}{3}\frac{\pi r^3 \rho}{\varepsilon_0}$$

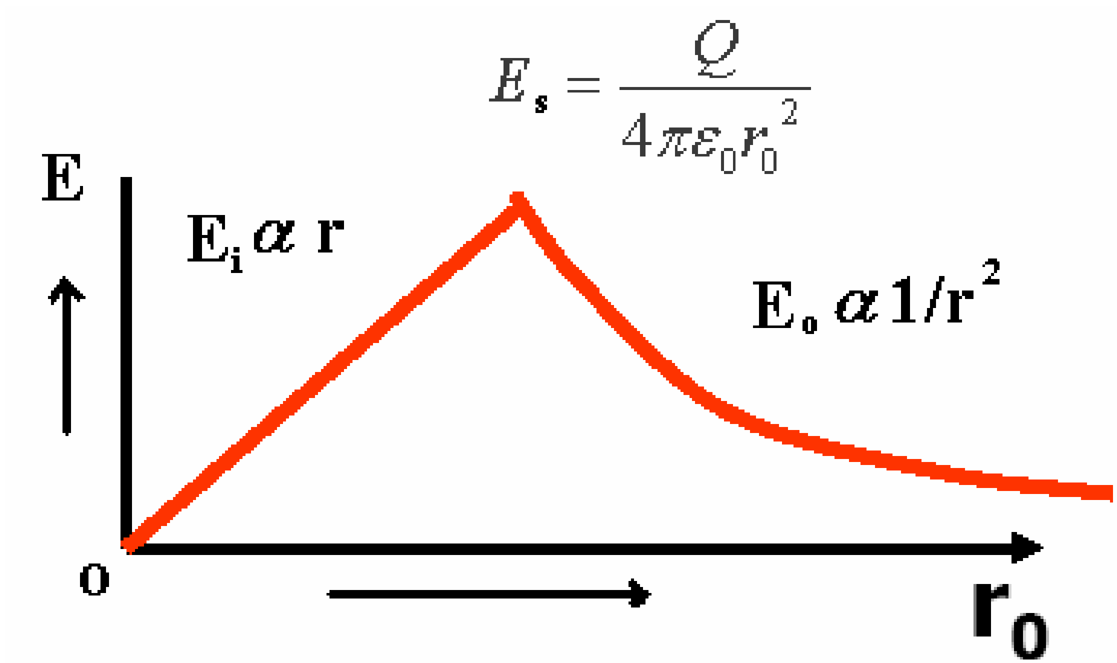
πε

$$E_i = \frac{1}{4\pi\varepsilon_0 r^2}\left(\frac{4}{3}\pi r^3 \bullet \frac{Q}{\frac{4}{3}\pi r_0{}^3}\right)$$

$$E_i = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{r_0 3}$$

$$E_i \, \alpha \;\; r$$

The variation of electric field strength with the distance from the center of spherical symmetric charge distribution



(iii) Electric Field due to a concentric spherical shells

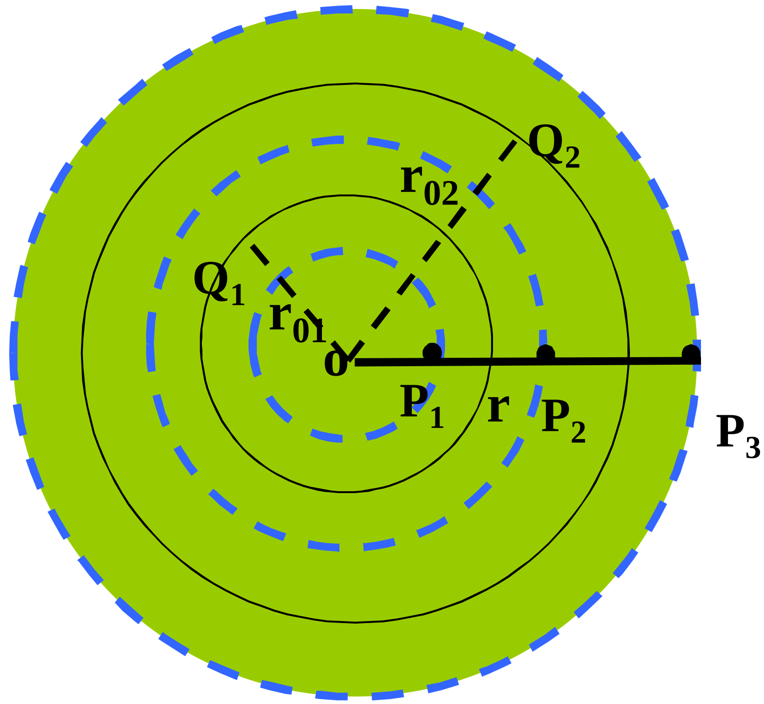
Two concentric spherical shell of radii r_{01} and r_{02} meters bearing charges Q_1 and Q_2

(a) Field inside the inner shell $r < r_{01}$

Charge inside the shell of radius r is zero

∴ Electric field intensity at P_1

E = 0



(b) Field between the shells $r_{01} < r < r_{02}$

Charge inside the shell of radius r is Q_1

∴ Electric field intensity at P_2

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r^2}$$

(c) Field outside both shells $r > r_{02}$

Charge inside the shell of radius r is $Q_1 + Q_2$

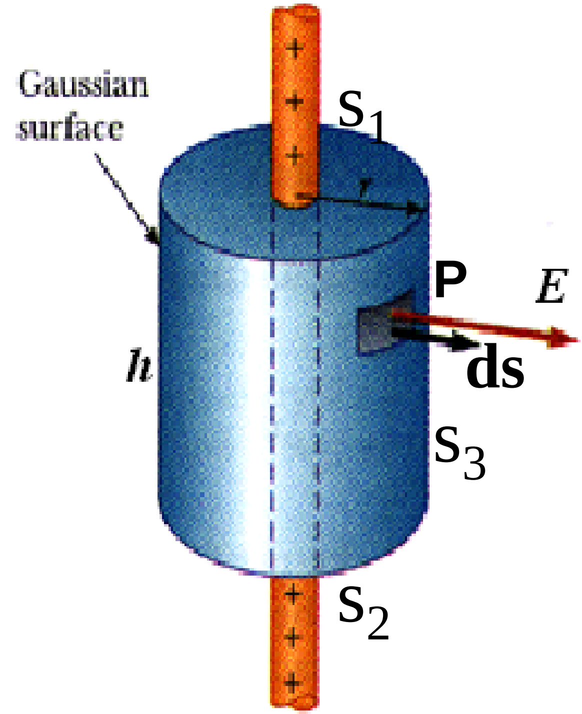
∴ Electric field intensity at P_3

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 + Q_2}{r^2}$$

(b) Cases of cylindrical symmetry

(i) *Electric Field strength due to infinite line charge*

- Let us consider an infinite line of positive charge with a linear charge density $\lambda = q/h$
- We wish to find the E field at a distance r from the line
- Let us now enclose this line with a cylindrical Gaussian surface



- There are three surfaces to consider. The upper (s_1) and lower(s_2) circular surfaces have normals are perpendicular to the electric field, thus contribute zero to the flux.

The electric flux due to each plane faces $\oint_{s_1, s_2} E \cdot ds = 0$

The electric flux due to curved surface $\oint_{s_3} E \cdot ds = E.2\pi rh$

Now according the Gauss law

$$\varepsilon_0 E \oint dA = q$$

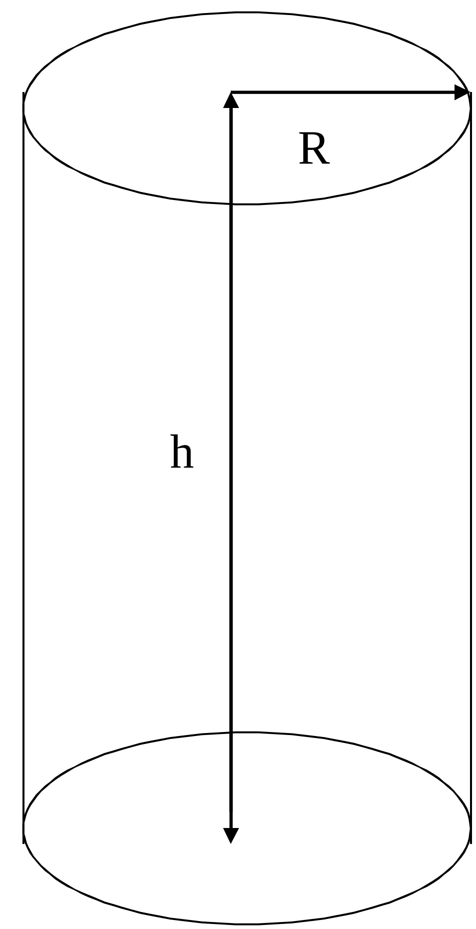
$$\varepsilon_0 E(2\pi rh) = \lambda h$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Thus the electric field strength is inversely proportional to r.

(ii) ***Electric Field strength due to a uniform infinite cylindrical charge***

Let us consider that electric charge is uniformly distributed within an infinite cylinder of radius R



If q is the charge per unit length and ρ is the volume charge density, then for a cylinder of length h and radius R

$$\pi R^2 h\rho = qh$$

$$\rho = \frac{q}{\pi R^2}$$

Case (i) When point lies outside the charge distribution i.e. r >R

Due to symmetry the electric field strength E_o is every where normal to the curved surface

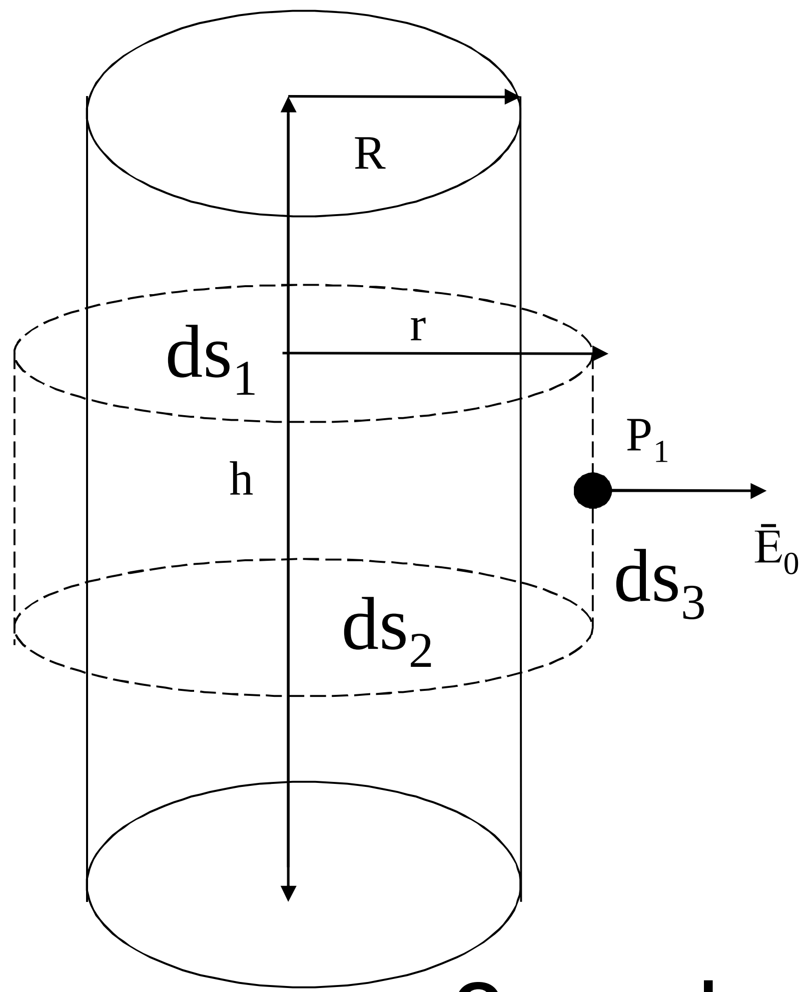
Further E_0 being parallel to two flat bases of the cylindrical surface considered, the contribution to electric flux due to circular surfaces is zero.

∴ Electric flux through the cylindrical surface assumed

$$\int_s E \bullet ds = \int_{s_1} E \bullet ds_1 + \int_{s_2} E \bullet ds_2 + \int_{s_3} E \bullet ds_3$$

$$\int_s E \bullet ds = 0 + 0 + \int_{s_3} E_0 ds_3 \cos 0^0$$

$$\int_s E \bullet ds = E_0 \int ds_3 = E_0 \cdot 2\pi rh$$



Q_{encl} = qh

According to Gauss's theorem

$$\int_s E \bullet ds = \frac{Q_{enc.}}{\varepsilon_0}$$

$$E_0 \cdot 2\pi rh = \frac{qh}{\varepsilon_0}$$

$$E_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{r}$$

Thus the electric field strength due to a uniform infinite cylindrical charge at any point outside the charge distribution is same as that due to an infinite line charge.

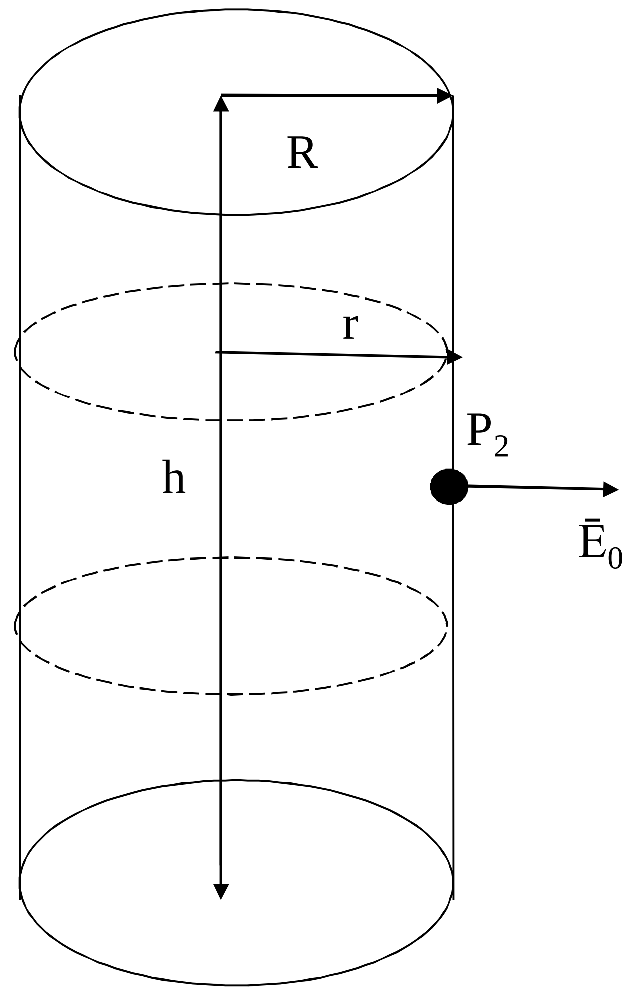
Case (i) When point lies on the surface of charge distribution i.e. r = R

In this case according to Gauss's theorem

$$\oint_s E \bullet ds = \frac{Q_{enc.}}{\varepsilon_0}$$

$$E_s \cdot 2\pi Rh = \frac{qh}{\varepsilon_0}$$

$$E_s = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{R}$$



Case (i) When point lies inside the charge distribution i.e. r < R

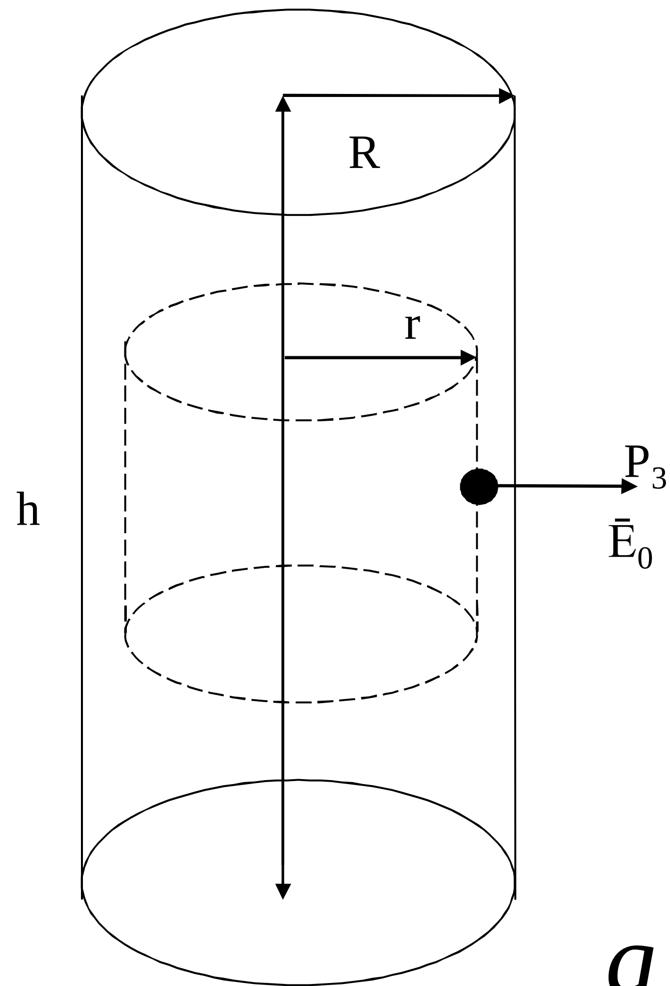
According to Gauss's theorem

$$\int_s E \bullet ds = \frac{Q_{enc.}}{\varepsilon_0}$$

$$E_{in} \cdot 2\pi r h = \frac{\pi r^2 h \rho}{\varepsilon_0}$$

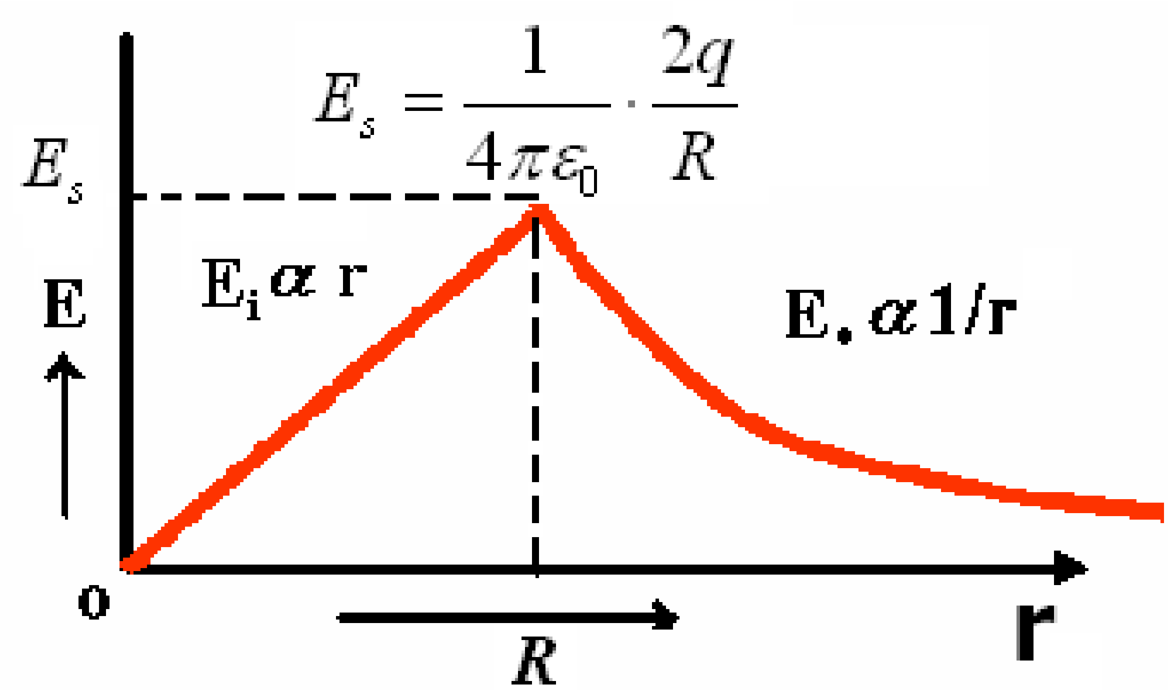
$$E_{in} = \frac{\pi r^2 h \rho}{\varepsilon_0 \cdot 2\pi r h} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2qr}{R^2}$$

$$E_{in} \propto r$$



$$\rho = \frac{q}{\pi r^2 h}$$

The variation of electric field strength E with the distance r from the axis of the uniform infinite cylindrical charge distribution



Electric field strength due to an infinite non-conducting flat sheet of charge

- Let us now consider portion of nonconducting (*Insulator*) sheet of charge having a charge density σ (charge per unit area)
- Consider an imaginary cylindrical Gaussian surface inserted into sheet
- The charge enclosed by the surface is $q = \sigma A$
- Due to symmetry electric field strength E is normal outwards at the points on the two plane surfaces and parallel to the curved surface.

➢ There is no flux from the curved surface of the cylindrical

➢ There is equal flux out of both end caps

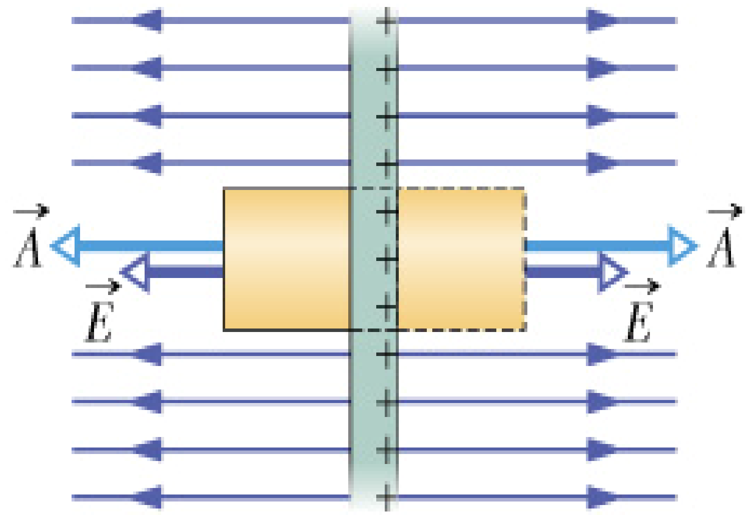
∴ Total electric flux = EA + EA = 2EA

According to Gauss theorem

$$\oint E \bullet dA = \frac{Q_{encl}}{\varepsilon_0}$$

$$2EA = \frac{1}{\varepsilon_0}(\sigma A)$$

$$E = \frac{\sigma}{2\varepsilon_0}$$



Thus electric field strength due to an infinite flat sheet of charge is independent of the distance.

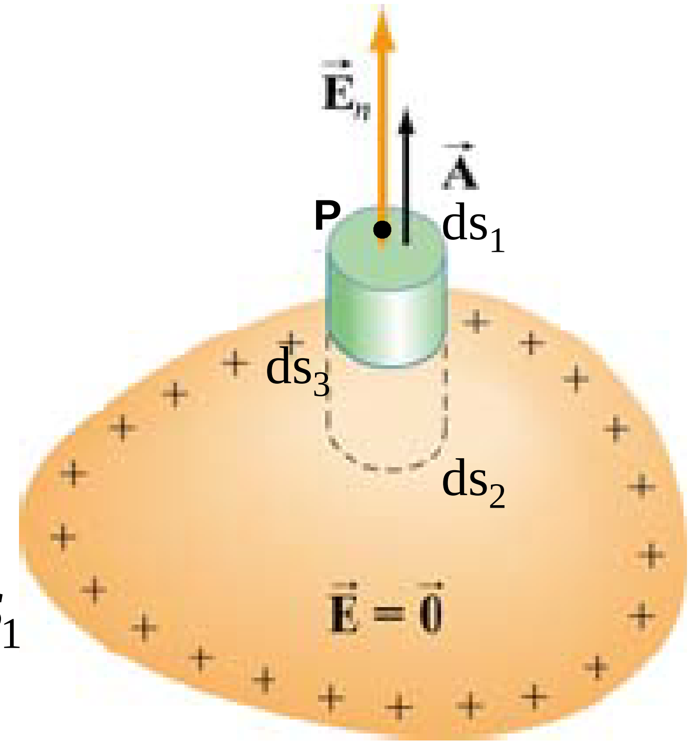
Electric field strength just outside a charged conductor

Consider a small Gaussian cylindrical box as drawn in fig.

Let the surface charge density on the surface of the conductor be σ

Let the area of each base is --- a

The Electric field inside the conductor is zero



Total electric flux

$$\int_s E \bullet ds = \int_{s_1} E \bullet ds_1 + \int_{s_2} E \bullet ds_2 + \int_{s_3} E \bullet ds_3$$

$$\int_s E \bullet ds = \int_{s_1} E \bullet ds_1 = \int_{s_1} E ds_1 \cos 0^0 = \int_{s_1} E ds_1$$

$$= Ea$$

Charge enclosed by the cylinder **Q $_{\mathbf{encl}}$ = σ.a**

According to Gauss theorem

$$\oint E \bullet ds_1 = \frac{Q_{encl}}{\varepsilon_0}$$

$$E.a = \frac{1}{\varepsilon_0}(\sigma a)$$

$$E = \frac{\sigma}{\varepsilon_0}$$

The electric field strength at any point close o the surface of a charged conductor of any shape is equal to 1/ε_0 times the surface charge density σ